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This volume is one of the fruits of a 3-year research project, *Space and Time After Quantum Gravity*, funded by the John Templeton Foundation.¹ Our goal was to explore the idea that attempts to quantize gravity either significantly modify the structures of classical spacetime, or replace them – and spacetime itself – altogether. It is a premise of our work that philosophy and physics are intertwined, so that advances in physics entail revisions in philosophy, but also require conceptual – that is, philosophical – advances and refinement. Hence our project activities were focussed on bringing interested physicists and philosophers into conversation.

Thus, in addition to their research, project members organized numerous colloquia, workshops, and schools, and ran three essay contests: our work is archived at www. beyondspacetime.net. From the researchers who participated in these events we selected a group that represents the cutting edge of a range of topics concerning the nature of spacetime in the new physics of quantum gravity, and invited them to contribute to a pair of volumes. One – *Philosophy Beyond Spacetime* (Wüthrich, Le Bihan, and Huggett, forthcoming) – deals more directly with the implications of quantum gravity for traditional philosophical concerns. This volume deals more with questions that require philosophical analysis, arising in the development of different approaches to quantum gravity. This distinction is a somewhat hazy one; several articles could have fitted equally well in either volume. But roughly speaking, the former volume should interest a wider range of philosophers, and the present volume a wider range of physicists (also being the more technical of the two); but physicists and philosophers with interests in our foundational questions should find both volumes valuable.

Even with two volumes, we could only select a small proportion of the researchers who were involved with the project, and not every topic, and far from every speaker, could be included here. So we have attempted to select a representative collection of papers that cover (i) research in the most active foundational areas in the field, and (ii) a range of approaches and questions within each topic. We hope, then, to provide a fairly comprehensive snapshot of the state of the field, to encourage further dialogue between physics and philosophy, and to promote further work.

The chapters in this volume are organized around three main themes: the possible 'emergence' of spacetime, the role of time in quantum gravity, and more specific interpretational

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issues raised by quantum gravity. The remainder of the introduction sketches these themes and the contributions. These sketches focus on some (not all) important ideas in order to show how the papers develop common themes from different angles – they are not intended to replace reading the chapters, which contain much more than can be discussed here! Rather, we hope that the sketches will whet the reader's appetite for what follows.

1. Spacetime Emergence

The first section addresses the question of how the classical spacetime of general relativity (GR) and quantum field theory (QFT) might be derived or emergent in theories that attempt to quantize gravity: we shall say 'quantum theories of gravity' (QTG) in order to be clear that the category includes any approach that aims to unify gravity and the quantum (and not only those that attempt to apply quantization strategies to GR). One question is the different senses in which classical spacetime might be derived from, or emerge from, or reduced to a more fundamental theory, without the full structures of classical spacetime. Another question approaches the issue diachronically, asking whether classical spacetime could have been 'created' from something non-spatiotemporal at the big bang.

A traditional framework for thinking about the derivation of classical spacetime is given by the 'Bronstein cube' (Bronstein, 1933; see figure ?? of this volume), which can be thought of as picturing a system of physical theories as limits of one another. The dimensions are labelled with c, G, and \hbar , so that they represent non-relativistic, non-gravitational, and classical limits, respectively. The eight vertices are populated by various theories, for instance, Newtonian mechanics, special relativity and GR, and particle and field quantum mechanics); but of course the most significant vertex for our purposes is that occupied by a theory of 'everything' (or at least 'more') incorporating a QTG. QFT (in flat spacetime) can be found in the $G \to 0$ limit of this theory, and GR in the $\hbar \to 0$ limit). Put this way, the picture seems to embody a fairly straight-forward answer to the challenge of deriving spacetime; classical spacetime is an effective description of a QTG, which holds in a formal limit, and is a good approximation when the effects of the parameters in question can be experimentally ignored. But of course, that is much too quick (even for known theories): what is the theory? Does the parameter actually appear in a way that lends itself to taking such a limit? And what is the physical significance of the parameter in the theory, such that we can argue that we live in a regime in which it can be neglected? These, especially the last one, are not purely formal questions, but are the issues of interpretation that confront attempts to derive classical spacetime.

In the first chapter of this volume, Daniele Oriti argues that the cube in fact fails to capture an important formal and physical possibility, namely that the physical 'elements' or 'atoms' of a QTG may form spacetime only in special aggregate states, which have a spatiotemporal description in a large N, 'hydrodynamic' limit. In short, we need to add a fourth dimension, parameterized by the number of degrees of freedom, N, yielding a Bronstein-Oriti hypercube of QTG. Traditional programs for QTG start with the ordinary cube in mind: and so attempt either to quantize GR (as in the original loop quantum

gravity -LQG - program), or 'gravitize' QFT (as in the first string revolution). But, as Oriti points out, the renormalization group revolution in statistical mechanics has yielded a formal and conceptual understanding of large N systems that was not available to Bronstein in 1933. Moreover, as these programs have developed, they have started to indicate that the fundamental degrees of freedom may not be obtained by the direct approach of quantizing or gravitizing; for instance, string dualities can be interpreted as indicating some structure that 'quotients' the apparent differences in spatiotemporal structure between duals. Oriti surveys similar clues from other programs.

His 'fourth dimension' gives substance to the idea of spacetime 'emergence'. That is, if a set of physical quantities approximate those of a more fundamental theory in the limit in which a constant vanishes, there is a straightforward epistemic interpretation of the reduction: our observations are simply not fine-grained enough to be sensitive to perturbations arising from the parameter, in the circumstances. That is a simple, really quantitative, sense in which one theory reduces to another. A large N limit might be of the same kind, but as Oriti explains, it highlights another possibility, suggested by various concrete proposals. That is, that the atoms of the theory might be intrinsically non-spatiotemporal, and only take on a spatiotemporal aspect in suitable large N configurations. Note that for such a theory, the claim that the atoms are not spatiotemporal is not based on a direct interpretation of their degrees of freedom, but rather on the fact that they simply do not constitute spatiotemporal structure in all states; if they were intrinsically spatiotemporal they would have to constitute something spatiotemporal however they were configured. (Of course, this argument depends *in*directly on the interpretation: specifically on claims about how the atoms can be physically combined to produce spacetime.)

This possibility carries a deeper, qualitative kind of reduction, of spacetime from nonspacetime, and in the $N \to \infty$ limit, which underwrites a sense of 'synchronic emergence' often found in the literature. Though what is also generally expected is a formally and conceptually well-controlled map between the theories, constituting a 'reduction' in the classic sense, rather than the strong 'emergence' found in other parts of the philosophy literature.

As Oriti explains and illustrates, a theory in which atoms may or may not combine to constitute spacetime offers a further, even stronger sense of emergence. Namely, there may be the possibility of a 'transition' from a non-spatiotemporal to a spatiotemporal state, at the big bang perhaps: diachronic emergence, or 'geometrogenesis'. Indeed, the work of Oriti and his collaborators on 'group field theory' strongly suggests just this. Of course, geometrogenesis is formally and conceptually very puzzling, for the very concept of a 'transition' seems to imply time throughout the process, but by assumption there is no time 'before' geometrogenesis!

The possibility that the history of the universe includes the emergence of the temporal from the atemporal also arises in the second chapter, by Suddhasattwa Brahma. (The more general question of time in QTG is discussed in the chapters of the second section of this volume.) He presents results developed within the framework of 'loop quantum cosmology' (LQC), which implements high-level principles drawn from LQG. As such, it is to a considerable extent neutral on the nature of the 'atoms' of spacetime, their formal expression and conceptual significance: by assuming certain general features to be consequences of the underlying theory, LQC does not directly speak to the manner in which they are derived. The reasons for adopting such an approach are of course to obtain a framework in which concrete empirical consequences can be derived, without needing full knowledge of the fundamental theory or details of how to take appropriate limits; the results are assumed. While we cannot see a full story of emergence from studying such a theory, it is still a quantum theory, and, as Brahma explains, LQC does entail a significant result about the derived nature of spacetime. (Moreover, because LQC is based on general principles, the lesson holds of any theory that realizes them. Brahma argues that the results are not dependent on idealizing assumptions in the derivation – e.g., of sphericity – but follow from the physical principles of the theory alone.)

Specifically, the assumptions made appear to suffice for the resolution of classical singularities, at the big bang and (it seems likely) in black holes. Moreover, as Brahma explains, the resolution involves a transition from a Lorentzian metric signature in the classical region, to a fully Euclidean metric signature in the region of the singularity: without a change in the number of dimensions, there is spacetime classically, but only space in the quantum region! (As the chapter discusses, a similar idea occurs in the distinct context of the Hartle-Hawking 'no boundary' proposal (Hartle and Hawking, 1983).) We have in a sense the emergence of time from the non-temporal, but three points should be noted: first, we do not have full-blown geometrogenesis, because the quantum regime is not strictly non-geometrical, as in the cases Oriti discusses. Second, as a result, it is in principle possible to consider one of the spatial dimensions as that in which the spacespacetime 'transition' occurs, potentially providing the basis on which that issue can be resolved. However, one should not expect a well-defined Euclidean signature metric in the quantum regime resolving the singularity, but rather a 'fuzzy' one; so the situation is not straight-forward. To make progress on these questions, as Brahma discusses, one would need to open up the question of the 'atoms' of the theory: and perhaps in that case one would see that geometrogenesis does after all underlie the process.

Classical singularities are, of course, one of the consequences of classical physics of greatest interest in QTG. Not exactly anomalies in the sense of a failure of the laws (if one is prepared to accept manifolds with singular points removed); but places at which one expects a more fundamental theory to diverge substantially from GR, yielding novel predictions. For instance, a QTG might predict specific traces in the cosmic microwave background (CMB), or in the Bekenstein-Hawking thermodynamics of black holes. The first possibility is the topic of Robert Brandenberger's chapter.

He explains the nature and content of the CMB, and the conclusions about the origins of the universe that can be drawn from it using the 'theory of cosmological perturbations', in whose development he played a significant role. In particular, isotropy implies that today's Hubble radius is smaller than the future horizon of early points, while causality requires that currently observed structures were within the Hubble radius at early times. Moreover, there are two further criteria, inferred from the observed power spectrum of the CMB: first, acoustic oscillations require that the universe has been isotropic above the Hubble scale for a long time, and second, any theory of the early universe must explain the scale invariance

of the spectrum. Inflation is the conventional response to these constraints, but in the context of QTG it can only be an effective theory, to be understood in terms of some deeper quantum account of gravity. For instance, one might well expect to find a mechanism for inflation within string theory, but despite the efforts of theorists, no definitive mechanism has been found (e.g., Baumann and McAllister, 2015; see also Bojowald, 2002 for a proposed account of inflation within LQC). (Moreover, inflation is not without problems.)

However, as Brandenberger explains, there are alternative accounts of the early universe to inflation, which also satisfy the CMB criteria: these solutions typically try to take into account proposals for more fundamental physics, QTG. For instance, the initial singularity could be smoothed out with a bounce solution, in which the universe extends through the big bang into an earlier classical spacetime. Unlike the LQC bounce discussed by Brahma in the previous chapter, Brandenberger focusses on models based on string theoretic concepts, including the possibility of a T-dual universe on the other side of the big bang. While these proposals are speculative, they illuminate the way in which new physics in a QTG might resolve the puzzles of the CMB. Moreover, they again illustrate the idea that spacetime might be emergent in a diachronic sense, from a quantum state at the big bang; though again, whether they involve full geometrogenesis from non-spatiotemporal atoms depends on details of the scenarios that are not yet understood. (Some of the philosophical implications of this situation have been further explored in Huggett and Wüthrich, 2018.)

In the following chapter, Daniel Harlow returns to the question of synchronic emergence; the derivation of spacetime as an effective structure rather than its 'creation'. Specifically, he addresses two important lessons for QTG that black holes may be teaching us. He first argues that one can best understand the enormous difficulty encountered in quantizing gravity by considering the tension between GR and QM caused by the possibility of black holes. Specifically, a rod capable of measuring Planckian lengths must have a sub-Planckian position uncertainty, hence a minimum momentum uncertainty according to QM. Assuming a low (with respect to the speed of light) velocity, a minimum mass follows, which is easily seen to exceed the Planck mass. But a Planck length-sized object of mass greater than the Planck mass is inside a black hole according to GR, and incapable of measuring lengths. That is, black holes exemplify the difficulty in defining quantum observables for arbitrarily small regions in QTG.

Harlow goes on to describe how black holes help illuminate the nature of holographic duality (the latter is discussed further in chapters 11 and 12), and plausibly show the existence in string theory of the kind of synchronic emergence described by Oriti. Harlow makes the point that the duality is (if correct) an exact correspondence, holding between *fundamental* quantum theories on the boundary and bulk of Anti-de Sitter spacetime, known as 'AdS/CFT duality': a conformal field theory on the boundary and some form of string theory in the bulk. On the other hand, the bulk gravitational *field* arises as a derived, *effective* theory of the fundamental bulk quantum theory. The value of AdS/CFT duality is that the bulk quantum theory is not understood well enough to carry out such a derivation; but the boundary CFT is under enough control to allow the exploration of emergent gravitational – spacetime – features.

As Harlow points out, the philosophical literature has focused on the question of whether the duality between exact theories can be an asymmetric relation of emergence, generally concluding that instead it is some symmetric relation of physical equivalence. Harlow's central claim is that this focus ignores the fact that bulk spacetime physics is derived from boundary physics, and hence indirectly from exact bulk physics: an asymmetric relation of derivation.

Using a simple black hole model, in the formal framework of quantum information theory, Harlow goes on to illustrate how this relation is one of spacetime emergence. Briefly, three qutrits (states in 3-dimensional Hilbert spaces) live on the boundary, comprising a $3^3 = 27$ -dimensional Hilbert space of a boundary quantum theory. The effective bulk theory is represented by a single qutrit, living in a 3-dimensional subspace of the full theory; corresponding to the few degrees of freedom of a classical black hole. But the fundamental bulk theory is dual to that on the boundary, and so also lives in a 27-dimensional Hilbert space; what has happened to the other 24 dimensions? It's not at all surprising that the effective theory has fewer degrees of freedom than the fundamental; that's more-or-less what it means to be effective, in a general sense. Rather the question is, since these extra degrees of freedom are not those of effective bulk gravity, what bulk physics do they describe? Harlow's work indicates that they represent microstates of a bulk black hole within the fundamental bulk theory. This toy model then represents the situation envisioned by Oriti; one only has effective spacetime to the extent that the quantum state of the system has a component in the appropriate subspace – other degrees of freedom belong to a fundamental, non-spacetime theory. Insofar as the model accurately represents non-perturbative bulk string theory, AdS/CFT duality shows that that too is a theory of emergent spacetime.

2. TIME IN QUANTUM THEORIES OF GRAVITY

It has long been understood that a successful QTG could have significant implications for our understanding of the nature of time. Many of the difficulties – especially those related to the 'problem of time' – in constructing such a theory seem to stem from the tension between needing a classical time parameter in the dynamics, yet quantizing time by quantizing the metric. In this part of the volume we have collected four chapters that focus on time in the construction of QTG: what the implications might be, and how the conception might have to be changed in order to successfully quantize gravity.

The chapters draw on philosophical thought about the nature of time in this effort, showing nicely the interaction between the two disciplines. In particular, a central theme is the question of whether various QTG do or should realize a form of 'temporal becoming'. Physics typically views time from the point of view of analysis, in which quantities only 'flow' in the sense of taking on different values at different times, with rates understood as limiting ratios $\Delta f(t)/\Delta t$. This picture seems adequate, and indeed natural, in a classical spacetime background, since it mirrors the mathematical treatment of physical quantities. Traditional temporal becoming is the view that there is more to the passage of time than this picture captures: that later states are in some further sense *produced* by earlier ones,

or the later times are *created* after the earlier, or that successive presents *come to be*. The italics indicate that these terms don't merely redescribe the standard, analytic, account of physical time; what more they denote depends of course on the specific account offered (Savitt 2017 provides a survey). Becoming is also often combined with 'presentism', the view that in some substantive way the present is more real than other times: reality is becoming. Such a view is often contrasted with a 'block' conception of time, according to which the present is merely a matter of perspective, within a full space*time*.

These concepts are unpacked more fully in the following chapters, in relation to QTG. Three of them see becoming, in three different conceptions, as important to quantizing gravity. The final one takes an even more radical view: no becoming, but no block either, just (in some sense) a collection of frozen moments, fundamentally speaking, temporally unconnected.

First, Carlo Rovelli discusses how he thinks that spacetime – particularly time – should be understood in LQG (the chapter also includes a useful appendix summarizing the theory for non-specialists). He argues that a number of confusions regarding space and time arise because people mean different things when using the expressions 'space' and 'time'; he describes the concepts as 'stratified, multi-layered'. To counter these confusions, he distinguishes five senses of time (and parallel senses of space).

Relational time involves only the relations between events: the temporal position of one event is specified by temporal adjacency with the occurrence of another. This conception of time is common to many theories, including LQG. In contrast, *Newtonian time* is a fixed metrical structure, independent of the unfolding of events, and indeed of whether anything changes at all; it is exemplified by both Newtonian and special relativistic physics. Things are very different with the introduction of dynamical general relativistic time, which is understood in terms of clock time between events, which of course depends both on a dynamical metric and the path of the clock (as in the special theory as well). Rovelli explains how, with some subtleties due to quantum effects, this conception of time holds in LQG, thereby preserving what he takes to be an important lesson of GR.

In addition, he distinguishes *irreversible time*, connected with thermodynamics, statistical mechanics and the entropy gradient; and *experiential time*, our experience or feeling that time 'flows'. For Rovelli, these should be distinguished because they do *not* have any direct bearing on the nature of time from a distinctively LQG perspective, but have to do with statistical and neurological effects, respectively. They are thus distinguished because bringing them into the current discussion can sew confusion.

With these distinctions drawn, the chapter unpacks the notion of time in LQG, focusing on the importance of temporal becoming. First, while accepting that the relativity of simultaneity undermines an absolute 'present' and presentism – something challenged by Lee Smolin in his chapter – Rovelli argues that a block conception of time, devoid of becoming is not the inevitable consequence. Instead, he identifies the transition amplitudes of LQG with the coming to be of one state from another; moreover, since these are between spacetime states they can be further identified with regions of spacetime. Such a scheme does not require the global 'now' of classic presentism; but it does rest on becoming at a local 'here and now' and so is not a block universe picture either. Thus according to Rovelli, the choice between presentism and the block is a false one! In his view, time passes, things become, but locally rather than globally; this is the lesson for time from LQG. The remainder of the paper is devoted to showing in more detail how his interpretation of time (and the related understanding of space and spacetime) play out in LQG: the picture that emerges is one in which the universe is a 'network of quantum processes'.

One might ask whether his account of time leans more heavily towards a kind of 'local presentism' or a 'block universe'. The answer depends on how one fleshes it out. On the one hand, the system of transitions that make up a universe has something of the structure of block universe. However, Rovelli rejects questions of whether all regions or just the herenow is real, as a merely conventional one about the definition of 'real'. On the other, if describing quantum transitions as becoming is not merely verbal, but denotes some strong ontological status, then the view is more sympathetic to presentism. Here Rovelli's view of experiential and irreversible time (and his (2017) view that it may be perspectival) suggests that he does not subscribe to a 'thick' notion of becoming either.

We now turn to the chapter by Fay Dowker, which also addresses the question of temporal becoming and the block universe, but in the context of causal set theory (CST), which she argues realizes temporal becoming in a strong ontological sense, against defenders of a block universe. According to CST, the universe is constituted by a 'casual set', a discrete structure consisting of elements with causal or temporal relations between them; the manifold picture of spacetime used in GR is an approximation, applicable in some regimes. As the theory currently stands there is no full quantum version of the dynamics, instead what is given is a classical but stochastic description, that gives rise to 'classical sequential growth' models according to which the causal set grows dynamically – 'becomes' – by the addition of new elements. Dowker sees the births of new events as something that objectively happens, underwrites the irreversibility of time, and that moreover could be a physical underlying objective process that explains experiential time. In other words, her account of becoming not only has a different source to Rovelli's, she argues that quantum gravity has implications for conceptions of time that Rovelli thinks it does not.

Now, traditional conceptions of time 'flowing' or 'becoming' typically rely on some form a global present, either in presentist or growing block accounts. The problem is of course that a global spacetime present allows for an objective time parameter and so is not generally covariant; this seems to be an undesirable step backwards towards a pre-relativistic understanding of time. Dowker argues that CST, however, provides an alternative model of flow without such a global now. Elements of the causal set are only objectively created before or after one another when they are in each others' causal pasts or futures; but there are no facts of the matter about the order in which elements that are not causally related were created. In turn, this is encoded in the equality of transition probabilities for paths that reorder the creation of such elements. The resulting temporal becoming is what Rafael Sorkin (2006) has dubbed 'asynchronous becoming', a localized form of becoming in a multiplicity of 'nows'. (An earlier version of Dowker's proposal has been critically discussed in Wüthrich and Callender, 2017.)

Dowker argues that CST can thus accommodate both 'being' (the baby) and 'becoming' (its birth) – unlike the block universe, which fails to capture the latter aspect. In this

manner, it reconciles two sides of a long-standing debate about which of these features ought to be given priority by embracing the essence of both. Arguably, one would expect *being* to refer to the objective structure of spacetime or of a causal set, while *becoming* would be rendered subjective by virtue of being relativized to a frame or a worldline. Surprisingly, Dowker defends the opposite view that being is subjective, whereas becoming is objective. According to her, the birth process of an atom of spacetime, and hence the becoming, is independent of any observers or frames as it constitutes an objective physical process. Conversely, there is no objective world of being; being is derivative in that it depends on a prior process of 'birthing', and what is objective is only each atom's past as that is what has become as of this atom. Thus, being is relative to each atom and in this way 'subjective'. Finally, Dowker asserts that this view of 'asynchronous becoming' is only possible in a discrete spacetime, and hence not available in GR.

In the next chapter, Lee Smolin lays out the philosophical framework of his work over the past 20 or more years, a research program aimed both at providing a realist interpretation of quantum mechanics and at quantizing gravity. (While the chapter discusses how the two aims are intertwined, here we will focus on the latter.) At the foundation of this work is a commitment to an aspirational form of 'relationalism': a methodological imperative (rather than *a priori* truth) to seek to remove arbitrary – 'absolute' – elements from physical theories. In part, he sees this principle in the history of science: eliminating absolute spacetime structure, including point identity in favor of equivalence-up-to-diffeomorphism, to give one example amongst the many he presents. In part, he sees it as guiding the search for a QTG.

Also central to his program is a form of temporal becoming that privileges time over space, a view developed in earlier publications (e.g., Smolin, 2013, and Smolin and Unger, 2014). He does not use the expression 'temporal becoming' explicitly, but his description of the view is clear: "the aspect of time I assert is irreducible is its activity as the generator of novel events from present events... The thick present is continually growing by the addition of novel events. At the same time other events in the thick present, having exhausted their potential to directly influence the future, slip from the present to join the always growing past."² Moreover, the present, but not the past or future, is real, so this is a form of presentism. Smolin's language expresses the difference between the traditional physical conception of time based on the reals, and that appropriate to the dynamics of his approach; indeed, he explicitly rejects the possibility of the kind of fixed, deterministic dynamical laws typified by the familiar differential equations of physics. Instead, things do not just happen one after the other, but each (irreversibly) 'generates' the next. Thus Smolin proposes a stronger conception of 'becoming' then Rovelli's; one similar to that of causal set theory, explored by Dowker.

Of course he is aware of the challenges to the 'present' presented by the relativity of simultaneity, but he points to the theory of 'shape dynamics', to demonstrate that they can be addressed. This classical theory (its quantization is discussed by Gomes in the following chapter) is based on preferred spatial slices, but is locally indistinguishable from

²The present is 'thick' in the sense that it contains events that are causally related.

general relativity. Although shape dynamics is relational, as we shall see in the next chapter it is most naturally thought of as a theory in which space is fundamental, and time derived: an example of 'timeless relationalism'. As we noted, instead Smolin advocates the primacy of time over space: 'temporal relationalism'. (Clearly this sense of time is stronger than Rovelli's 'relational time'. Extending Smolin's terms, Rovelli would seem to advocate a form of 'spacetime relationalism', taking neither space nor time as more fundamental.)

In addition to time and causation (or generation), Smolin also proposes that energy and momentum are fundamental: the fundamental states live in momentum space rather than physical space. Even at the classical level, spacetime can be reconstructed, exemplifying the sense in which space can emerge from time and the non-spatial. (To editorialize: insofar as momentum space is non-spatial.) On quantizing, one discovers that locality – meaning point coincidence of trajectories in this case – is relative: as simultaneity is motion dependent, locality is energy dependent. For Smolin, this result shows that spacetime itself is as observer-dependent as simultaneity, an effective construct in limited regimes.

These ideas provide a cohesive conceptual framework for the program of temporal relationalism, and a number of the contributions that Smolin and others have made to quantum gravity. The final section of his chapter gives a comprehensive overview of the program and what has been accomplished, in the light of this framework. We will just emphasize the more recent developments from his study of 'energetic causal set models', which directly implement the tenets of temporal relationalism: from an underlying irreversible non-spatial dynamics, a reversible particle dynamics in a Lorentzian spacetime emerges as an effective structure. Moreover, the Einstein field equation can be derived from the thermodynamics of the model. Thus, energetic causal models exemplify the way in which *spacetime* might emerge from a theory of temporal becoming.

In the final chapter of the section, Henrique Gomes investigates the picture of time arising from his recent work on the foundations of QTG, drawing a radically different picture. The first half discusses a train of thought that points to the version of shape dynamics developed by Gomes and his collaborators (Gomes et al 2011). Tracing the root of the problem to the difficulty in quantizing the causal structure of relativity, he sketches the challenges facing QTG: the 'problem of time' for Hamiltonian approaches, and the problem of parameterizing the space of 4-dimensional Lorentzian spacetimes for covariant approaches. In both cases, Gomes notes that resolutions can be found in the space of classical solutions ('on shell'), but that this is inadequate to a quantum theory; he draws the lesson that causal structure should not be built in to a fundamental quantum theory, but recovered in an effective theory.

Gomes argues that shape dynamics implements this idea classically by replacing space time symmetries with spatial symmetries: 'position relationalism' and 'scale relationalism' (which are the unique symmetries acting solely on configuration space, rather than on phase space). But, he asks, how are we to understand time – the *dynamics* of shape dynamics – in the theory? The original, classical theory proposed the simplest solution: introduce an independent time parameter. To quantize, one could use this parameter to define a Schrödinger equation. But in the current chapter Gomes is dissatisfied with that approach: such an

absolute time violates the relationalism that he sees as the cure for the problems of quantizing spacetime structure. In this then, he takes the opposite interpretational view to Smolin.

Instead, Gomes applies to time the idea that classical structures need only be recovered effectively. In the latter part of the chapter, he develops a formal framework to realize this idea, and develops an interpretation with profound consequences for time. The formalism involves a notion of quantum path integral that is independent of a time parameter, and a measure over configuration space, which lead to a Born Rule for the theory. This framework involves selecting a privileged 'in state', which Gomes notes is key to the notion of a 'record'. Concretely, it allows him to define a formal notion of a record, the *apparent* trace in the present, of a past transition from the initial to the present state. Significantly, this definition requires a semi-classical approximation, and so such records are inherently semi-classical.

His interpretation of this framework is that fundamentally there is no duration, only instantaneous configurations; and all possible ones are on an equal ontological footing, akin to a many-worlds interpretation. Fundamentally, there is no time at all. Like Barbour (1999) but unlike Smolin, Gomes thinks that the past of any configuration is fully reducible to the records (or 'time capsules') contained in that configuration; however, unlike Barbour, Gomes' time can only be reconstructed effectively, at the semi-classical level, since records are semi-classical. In sum, there is effective time, but only an instant. An extended past can be projected from the effective apparent records held by that instant, but it is merely a 'just-so' story: even effectively, only the instantaneous records themselves are real. In this view space is fundamental, and time barely real, and there is neither passage, nor the block.

Of course, this image of time diverges greatly from our ordinary conception, and in particular clashes violently with our concepts of personal history: an issue that Gomes also takes up.

In this part of the book we have thus seen four different responses to how we should understand time in QTG. Some authors – in particular Dowker and even more strongly Smolin – award to time a more pronounced and special role than to space, in a departure from the orthodox understanding of relativity. In contrast, we take Rovelli to advocate a view in which space and time are on more equal footing – that is by both being emergent from the underlying quantum theory in basically the same way. Finally, Gomes takes space to be the more fundamental aspect of reality.³

3. Issues of Interpretation

The first two sections of this book investigated in various ways the implications of QTG for the nature and emergence of space and time, but such theories have raised other important questions for the interpretation, epistemology, and metaphysics of science, some

 $^{^{3}}$ For a similar observation about the diverging views on the nature of time that can be found among researchers working on developing a QTG, see Matsubara (2017).

of which have been hotly debated. The chapters of this section address several such issues: the 'information loss' paradox(es); the meaning of string theory's dualities; and the implications of QTG for the logic and metaphysics of possible worlds.

First, what is the physical nature of black holes? Often thought to be a key to quantum gravity, black holes appear to admit a thermodynamic treatment, suggesting that – perhaps – a QTG ought to provide a description of its microstates. The issue of black hole thermodynamics is taken up by David Wallace in his contribution. He argues, contrary to recent philosophical criticisms (Maudlin, 2018), that black hole 'information loss' (the failure of temporal reversibility, manifested as non-unitarity in QM) is indeed paradoxical, though not in the way often presented. Overall, the point is that one expects to understand the Bekenstein-Hawking entropy of a black hole in Boltzmannian terms as the logarithm of the fundamental microstates; and indeed Hawking radiation (effectively) as a decay channel of the fundamental state. But this picture relies on treating a black hole both as possessing thermodynamical properties such as temperature, and being quantum mechanical, and these features come under pressure in the information paradox.

Popular presentations present the 'paradox' as an issue for the *endpoint* of evaporation, at the 'Hawking time'. In starkest terms, a black hole in a pure state undergoes unitary evolution (Hawking radiation) until it is entirely gone, and all that remains is thermal radiation, a mixed state. Yet it is a mathematical impossibility for a unitary process to turn a pure state into a mixed one. But as has been pointed out for over 20 years (and recently insisted upon), because of the classical singularity the endpoint of the process is not a Cauchy surface, and so insisting on a deterministic, unitary evolution is at best to make a substantive, controversial claim. However, Wallace explains that the emission of thermal radiation from a black hole produces a paradox in the physical principles believed by many (though not all) to describe the process – well before the Hawking time, or even without complete evaporation (and so is not resolved by considering the endpoint).

On the one hand are statistical mechanical principles. The discovery of Hawking radiation elevated the Bekenstein thermodynamical description from analogy to reality, by showing that the description remains valid when black holes interact with other thermodynamical systems: in particular, they can exchange heat. Since thermodynamics in general is understood in terms of a microphysical description – with entropy as the logarithm of microstates – one concludes that the same is true of black holes: that black hole thermodynamics describes, in the large, the statistics of black hole microstates. (Hence the programs to derive, in string theory and LQG especially, the entropy and radiation spectrum of black holes from posited microstates.) In particular, as Wallace explains, the black hole is often modeled in the statistical mechanical 'membrane paradigm' as a surface located around the horizon, containing the microphysical degrees of freedom, which are transformed to thermal radiation and lost – decreasing their Boltzmann entropy. Whatever the underlying theory, if the microstates are quantum, this process must be unitary. (It is worth emphasizing that the membrane paradigm is most popular within string theory, which is the real target of Wallace's chapter.)

On the other hand, derivations of Hawking radiation rest on the principles of QFT in curved spacetime. These extend the well-tested principles of QFT in flat spacetime, but

do not concern situations in which a full quantum theory of gravity is needed, of extreme curvature or energy density. They entail that Hawking radiation is in a thermal state, a mixture with unentangled modes. As noted, a unitary evolution cannot produce a mixed state from a pure one, so modes of the Hawking radiation are understood as entangled with degrees of freedom of the black hole, which are traced out in the usual way when one observes an entangled subsystem.

But there is a limit to this understanding: according to the statistical mechanical membrane paradigm that Wallace endorses, at a certain point – the 'Page time' – the decreasing Boltzmann entropy of the black hole means that there are no longer degrees of freedom with which the thermal modes can entangle. Yet the principles of QFT used in the derivation of Hawking radiation entail the continued production of a mixed state – in violation of unitarity, and so of a quantum description of the situation.

As Wallace reviews, the Page time is much shorter than the Hawking time for complete evaporation (roughly half), and the scenario can even be modeled in evaporating black holes without complete evaporation. So the qualms about the popular form of the information paradox do not arise; Wallace's explication of the 'Page paradox' is thus much sharper, apparently calling for giving up either QFT in curved spacetime or the membrane paradigm. Neither option is attractive (at least within string theory): the former seems to imply that quantum gravitational effects are relevant whenever spacetime is curved, not only when the curvature is Planckian; the latter threatens to make black holes an exception to the statistical mechanical approach to microphysics. Wallace's essay reviews in detail how tight and difficult a bind this is; and how attempts to resolve it with appeals to 'black hole complementarity' seem to lead to the 'firewall paradox'.

In the second chapter of this section, Richard Dawid discusses the current and future status of string theory. He makes an argument to the effect that the difficulty in giving a complete description of string theory – its 'chronic incompleteness' he calls it – can be explained if string theory is a 'final theory'. (It should be noted that Dawid uses the expression 'string theory' in a wide sense, including future developments of the theory and other associated ideas including M-theory.) He presents a number of observations suggesting that string theory is a likely candidate for being a final theory of physics: first there is the universality of string theory, that is it does not seem to be restricted to describing only a certain class of phenomena. Second, string theory does not have any fundamental dimensionless free parameters. Third, string theory has a minimal length scale.

Dawid sees these three features are in stark contrast to past physical theories. First, past physical theories have been expected to only be applicable to a restricted domain of phenomena. Second, typically our theories have had freely adjustable dimensionless parameters that could be chosen to fit what we observe in nature. The values of these parameters may be explainable by a more fundamental theory or by the way in which the theory is embedded in a specific physical background. The lack of such parameters is taken by Dawid to suggest that string theory is a final theory. Finally, our previous theories have been thought to be replaceable and seen as effective theories arising from more fundamental theories that are valid at smaller length scales. With the minimal length scale of string

theory – suggested by the T-dualities that allow a small scale to be eliminated in favor of a description in terms of larger distances – Dawid argues that there is no reason to expect string theory to be similarly replaced by a more fundamental theory.

Just as he sees these features pointing to string theory as a final theory, he also argues that they make it hard to fully understand and articulate, leading to chronic incompleteness. Central to his argument is the way in which string theory is primarily understood in terms of perturbative calculations around near classical limits. Such backgrounds themselves are put in by hand and are not part of the dynamical description. The absence of freely adjustable parameters in the theory – even though we seem to have many allowed groundstates – means that we cannot tune the theory to a classical state of some unknown more fundamental physics. Thus for a deeper description the backgrounds must be understood in terms of the non-perturbative dynamics of string theory itself. But while string dualities provide additional insights into the non-perturbative physics, they are not sufficient to this task. Broadly speaking, the challenge to developing such a deeper account of string theory is that it needs to handle situations that cannot be well approximated by any classical picture – thus making our common sense understanding of the situation even less useful for guiding us in the right direction.

Turning an apparent failure – chronic incompleteness – into evidence in favor of string theory as a final theory is at least controversial! But as with Dawid's earlier work, it should contribute to an ongoing discussion of what might characterize a 'final theory', and what research programs are worth pursuing. (And even whether it is worth pursuing such a theory, rather than taking smaller steps to greater but not final understanding.)

The next two chapters explore the interpretational and methodological significance of AdS/CFT (or gauge-gravity) duality mentioned above – the core of recent research in string theory – in two different ways. (Daniel Harlow addressed its significance for spacetime emergence in his chapter, discussed above.) First, Sebastian De Haro addresses the relation of duality to 'physical equivalence', and the implications of such equivalence. This is a topic that has been addressed by philosophers, but De Haro provides a rigorous framework for the conceptual situation that permits clear and precise answers to the important questions – including whether spacetime is primitive or derived in string theory. Whereas Harlow raised the question of whether one side of the duality was *derived* from the other, here the issue is different: when are duals *equivalent*, and what follows if they are?

De Haro defines duality in general to be a formal relation between theories, a partial isomorphism of a certain kind: more specifically, an isomorphism between parts of two theories, with sufficient structure – a space of states, set of observables, and dynamics – to themselves be called 'theories'. So for instance, in the case of a pair of simple harmonic oscillators (SHOs) with masses and spring constants related $(m, k) \rightarrow (1/k, 1/m)$, a duality maps $(x, p) \rightarrow (p, -x)$. A table of value-pairs over time would be the same if it displayed (x, p) for one system, or (p, -x) for the other; and so the duals share this common structure. And in general a duality picks out a 'common core theory', a part that the two theories share up to isomorphism.

Now, of course, for a real bob on a spring, the SHOs are still different systems since they have, *inter alia* observably different masses; it's just that if one were only given a

table of value pairs over time, one could not tell if they described (x, p) for one system, or (p, -x) for the other. The point is that 'physical equivalence' – intuitively, 'telling the same *story* about the world' – is not a purely formal matter, but also depends on what the duals *mean*, and whether they 'mean the same'. At one time the answer would simply have been 'if they make the same observable predictions', but part of the value of De Haro's contribution is to offer an account of the interpretation of theories that does make precise this distinction, without appeal to such crude verificationism.

The reason real dual SHOs are distinct is that in our world, their common core can be embedded in – or 'extended', in De Haro's terms, to – a larger theory that gives 'external' meaning to their terms: mass, spring constant, momentum, position. But one can envision a world in which the common core instead described *everything*: possible states are fully distinguished by the value-pairs, with the same allowed histories, and interpreted as the values of the only two canonical observables of the world. Then the duals are nothing but different tools for computing the dynamics, with their differences (in m and k, and x and p) as nothing but empty conventions needed to turn the mathematical handle. Then there would be no larger theory of the world (without uninterpreted surplus structure) in which the core could be embedded, so it could not receive an external interpretation, but rather an 'internal' one: like the interpretation we just gave, which simply maps the elements of the theory to their worldly referents. De Haro gives a precise account of the situation that will enable more focussed discussion of its consequences.

Given this framework, (at least) two interpretational issues remain. First, could we ever reasonably believe that the common core of a pair of duals was not extendable? That it captured all the physical structure of the world (in its domain), so that it was not just part of a broader (perhaps more fundamental) theory? As Jeremy Butterfield emphasizes, in a paper for the companion volume to this one (Wüthrich, Le Bihan, and Huggett, forthcoming), the formal existence of a duality alone does not show that. (De Haro does not claim otherwise.) Here one may be tempted to use methodological principles such as ontological simplicity to move from duality to 'unextendability'. Second, suppose that the world were such that the common core of a pair of duals indeed has no external interpretation: does it follow that the duals are physically equivalent? Perhaps instead they could describe a pair of worlds in which different physical quantities are instantiated in isomorphic patterns. This is not a question of physics, but metaphysics: how properties are identified across possible worlds. De Haro argues that the existence of two such worlds would violate the identity of indiscernibles. If one answers 'yes' to the previous questions for a pair of duals, then the duals are physically equivalent, with their content exhausted by an internal interpretation of their core.

In the final part of the chapter, De Haro applies his framework for duality to gaugegravity duality, showing that while the common core contains some weak spatiotemporal structure, it does not contain most of the structure of a spacetime theory. Thus if one answers the two previous questions positively, and adopts an internal interpretation of string theory then AdS spacetime is not fundamental, but merely a conventional description, and in that sense emergent. Next, in their jointly written chapter, Radin Dardashti, Richard Dawid, Sean Gryb and Karim Thébault address a number of questions regarding how one should think about the possible empirical consequences of such AdS/CFT-duality. While the original reason for studying the duality was to deal with quantum gravity and Planck scale physics, the formalism and mathematical results have more recently been used for dealing with other questions in physics than QTG: for instance, quark-gluon plasmas. The authors find it important to distinguish three different contexts in which an AdS/CFT duality could be applied, and want to explain why different conclusions would be warranted in the different contexts if the dual theories were shown to be empirically adequate.

In the first context, in which AdS/CFT-duality is used to describe fundamental physics (in a sense they explain), the authors argue that empirical success would not give us any reason to prefer one of the dual pictures over the other. Their argument is that each of the dual theories would be confirmed as much as the other in the Bayesian sense. Furthermore in the context of fundamental theories the authors do not think there is any good principled reason for assigning one of the dual theories with different priors to the other. The authors consider this as a good reason for not prioritizing one picture over the other when it comes to ontology. Either the ontological picture to which one should be committed is to be articulated on the basis of a structure that is shared between the two pictures – a common core – or one should accept some form of dual ontology where in some sense the ontology of both theories should be equally acknowledged. In both cases the upshot is that the duals are different descriptions of one single theory.

In the second context, the duality is supposed to relate effective theories for which there exists a single more fundamental theoretical description. In this context the situation is somewhat different and the authors open up the possibility that one of the dual pictures could justifiably be seen as a better description of the underlying physical reality: if the ontological picture suggested by one of the dual theories was closer to that of the more fundamental theory. In the effective context, arguments could also be introduced concerning whether or not one or the other picture could more plausibly be embedded in a larger, more encompassing description of reality. However, while the authors describe this possible way in which one of the duals could be given priority, they also point out that this way of reasoning relies on a rather strong form of scientific realism. Thus if weaker forms of scientific realism instead were assumed – where ontic commitments play a less central role – then the two dual theories would still be considered as on par and equally confirmed.

The third and final context is the instrumental one in which the duality is used for the purpose of making approximate predictions in another theory, where this other theory is not one of the two duals. The authors focus on the application of AdS/CFT duality to quark-gluon plasmas. These are governed by quantum chromodynamics (QCD), but the implications cannot be easily calculated because perturbation theory is not applicable in the relevant regime. However, it can be argued that a CFT would give results similar to those of QCD; then the CFT results can be calculated using the dual AdS description, to make *approximate* predictions for the plasma. These results are approximate because the duality with QCD is not exact, so that we have an example of the third context. The authors argue that the empirical success of predictions of this kind do not confirm

either of the dual descriptions; instead it only confirms the conjunction of QCD and the approximation scheme based on the duality that is used. Furthermore, there is no reason to take this kind of successful approximate prediction as constituting evidence supporting string theory as a QTG.

The final two chapters discuss the possible formalism of QTG, in particular relation to the metaphysics of 'possible worlds'. These of course play an important role in the thinking of such figures as Leibniz but, as Tiziana Vistarini explains in her chapter, especially in the modern modal logic of possibility and necessity. In the work of Lewis, for instance, one can interpret the claim that 'P is possible' in terms of an 'accessibility relation': as saying that P is true in some possible world accessible to the actual world. (For instance, if one defines a world to be physically accessible iff the laws of physics are true in it, then P is physically possible if there is some physically accessible world in which P is the case.) Moreover, from accessibility one can develop a graded relation of 'degree of similarity', so that we can talk of possible worlds being more or less similar. Lewis (1973) introduced this notion for an account of the logic of 'counterfactual conditionals': 'if P had been the case, then Q would have been the case' is true iff if in the most similar world(s) in which P is true, so is Q.

The manifold of possible worlds thus invoked, and the metaphysics of modality it represents, is thus derived from the logic of modal sentences; and so, as Vistarini explains, arguably on the logic of ordinary language, with inevitable imprecisions (for instance in the cardinality of worlds). The crux of her chapter is to argue that the moduli space of string theory – the space of possible models or theories – provides a physically grounded and metaphysically substantive extension of the space of possible worlds. She explains how this space has a topological structure, relative to a given model, induced by the space of deformations of that model, within moduli space. Crucially, she shows how this topology is strong enough to define a partial ordering on the points of moduli space, which she proposes interpreting as a similarity relation, and hence points of moduli space as possible worlds of the theory. This proposal of course raises a host of philosophical questions, which the chapter starts to address. For one thing, the space of worlds is precisely defined, allowing precise answers to questions of its structure: for instance, the similarity relation has a countable spectrum. For another, the worlds described by ordinary language – the 'manifest image' – should, in some way, be reducible to those of fundamental physics, including their structure of possibility; Vistarini sketches how such a reduction of modality might go in string theory, through a revised form of 'humean supervenience'.

Of course, many of the essays of this volume have proposed significant modifications to the classical spacetime picture of GR in a QTG. In the last chapter of this volume, Ko Sanders proposes another, using the tools of mathematical category theory to reformulate a classical spacetime theory, with an eye to a different route to quantization. Again, as in other approaches it is important that the classical picture can be recovered in appropriate regimes where we know that this picture is accurate.

As a starting point for the analysis, Sanders uses the framework of locally covariant quantum field theory (LCQFT). This framework is similar to algebraic quantum field theory (AQFT) but uses the tools of category theory for the purpose of encoding the features of locality and general covariance. In contrast to AQFT – where only one quantum system is given a description in terms of a C^* -algebra – LCQFT associates for each object in the category Loc of globally hyperbolic Lorentzian manifolds a corresponding object in the category Alg of C^* -algebras. In this axiomatic framework QFTs, can be formulated to take gravitation into account but without actually quantizing gravity; a way of formulating QFT in curved spacetimes.

In general terms, Sanders suggests that to go beyond LCQFT and to formulate a *bona* fide QTG one could try to preserve much of the structure used in LCQFT but replace the category Loc with another category. Using this other category the classical manifold description would only be a good approximate description in certain regimes. He does not propose such a category in this chapter, but proposes searching for it as a research program. Overall then, as with the papers in the first part of this volume, Sanders presents yet another example of a picture where the traditional picture of spacetime is an emergent and not fundamental feature of reality; this time using the framework of category theory.

The chapter also proposes in some detail that categories could serve as models of modal logic. More specifically he claims that the category Phys – whose objects are mathematical descriptions of physical systems – ought to be such a model. Here the possibilities that are modelled are not full possible worlds but rather physical systems, that could be extended to whole worlds but we do not have to deal with the whole worlds when articulating possibilities. Sanders argues that this aligns better with the actual practice of physicists since it is not typically the case that one needs to describe a full possible world when describing a physical system.

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