Homework Assignment 9

25 Science, Philosophy, and the Big Questions due 16 March 2012

For submission

1. (a) What experiment gives us good reason to think that light consists of waves? How does it lead to that result?

   (b) What experiment gives us good reason to think that high frequency light has its energy localized at points in space, like a particle? How does it lead to that result?

2. (a) What model of the atom tells us that electrons could be found anywhere in the vicinity of an atom's very small nucleus? On what physical theory is that model based?

   (b) How does the theory of atomic spectra suggest that the theory of (a) is wrong.

   (c) What theory of the atom results from taking the atomic spectra seriously?

3. (a) How does de Broglie's theory of matter waves connect the energy and momentum of particles with the frequency and wavelength of waves?

   (b) How does this theory make sense of the theory of the atom of 2.(c)?

4. Consider a wave packet used in de Broglie's theory to represent a particle. How is the particle's momentum affected if we make the spatial extent of the wave packet bigger or smaller? How does this difference relate to the 'Heisenberg Uncertainty Principle'?

   (a) What is the difference between interpreting the uncertainty of Heisenberg's principle as ignorance as opposed to indeterminateness?

5. What is the 'Schrödinger evolution' of a matter wave? What is 'the collapse of the wavepacket'?

6. In the standard analysis of the Schrödinger cat thought experiment, what leads to the definite survival or definite death of the cat?

For discussion in the recitation

A. Consider the sequence of theories that set us on the way to modern quantum theory. They mixed together components of classical physics with new quantum notions and, to use the 'old quantum theory' one had to invoke both classical and quantum notions at the same time:

- Planck's analysis of heat radiation assumed that heat radiation was generated by emission and absorbtion of light from classically described electric resonators. His analysis seemed to require that electric resonators only be allowed to adopt discrete energy levels, although classical physics told us that they could adopt a continuous range of energies.
- Einstein's 1905 light quantum hypothesis held that high frequency light energy is localized at points in space. Yet at the same time Einstein still allowed that interference phenomena were possible for light and that requires that the light be spread out in space.
- Bohr's 1913 theory of the atom took the classical theory of electron orbits in which electrons may orbit at any distance from the nucleus, but cannot do so stably. To it he added the assumption that these electrons can orbit stably, but only at very few discrete distances from the nucleus.

In all these cases, the theorists seem to make essential use of logically incompatible assumptions. Electrons cannot both be stable and not be stable, for example. The presence of a logical inconsistency is usually taken to be fatal to a physical theory. Yet here were successful theories that seemed to depend essentially on contradictory assumptions.

(a) Should we require our physical theories to be consistent?

(b) Do you know any examples of theories that were discarded when they were found to be based on contradictory assumptions?

(c) Are there other examples of successful theories that are based on inconsistent assumptions?

B. To sharpen the problems above, consider this. If a theory is contradictory, then it allows both the truth of some proposition A and also the truth of its negation not-A. In classical logic, one can deduce anything at all from a contradition. Here's the proof. (If you have had a logic class, this will seem entirely trivial. If not, you may be a bit startled by how easy it is to infer anything from a contradiction.) The inference combines two standard argument forms:

Addition

1. C
2. Therefore, C or D

Disjunctive syllogism

1. C or D
2. not-C
3. Therefore, D

To prove any proposition B from a contradiction (A and not-A):

1. A (Assumption)
2. not-A (Assumption)
3. A or B (From 1,2 by Addition)
4. B (From 2, 3 by Disjunctive Syllogism)

For example:

1. Electron orbits are stable. (Assumption)
2. Electron orbits are not stable. (Assumption)
3. Electron orbits are stable OR bananas are high in Potassium. (From 1, 2 by Addition)
4. Bananas are high in Potassium. (From 2, 3 by Disjunctive Syllogism)

What this tells us is that, in an inconsistent theory, we can deduce anything. So should we be so surprised that Planck, Einstein and Bohr can deduce their results from inconsistent premises? From inconsistent premises, we could deduce that planets orbit in squares; or that everything is made of licorice!

Or is there something more subtle at work? Planck, Einstein and Bohr seem to have found some deep truths about the world. How can they be extracted from the snake pit of logical inconsistency?

C. Quantum theory is an indeterministic theory. That means that a complete specification of the present state of some atomic system does not fix its future. Here's how we apply this idea to radioactive decay. If you have a single atom of Neptunium NP 231 93, there is a one in two chance that it will decay over the next 53 minutes. According to standard quantum theory, that is all you can know. There is no way to know ahead of time whether the atom will decay. Do you really believe that? Might it be if we had a more complete picture of the compicated, hidden recesses of this atom that we'd see some tiny difference between those atoms that end up decaying and those that do not? Ought we expect some future theory of the insides of atoms to tell us about these sorts of hidden properties? Ought we to demand such a theory before we can say we really understand radioactive decay? Or should we comfortable with the idea that some processes just are indeterministic?

D. To get a sense of how the Heisenberg uncertainty principle applies, consider the problem of balancing a pencil perfectly on its tip. Here is what is needed for success in the balancing operation: you have to align the center of mass of the pencil exactly over the pencil's tip; and, as you take your fingers off the pencil after doing this, you need to leave the pencil perfectly at rest. What does Heisenberg's uncertainty principle tell you about your chances of success?

E. The 'measurement problem' remains a lingering difficulty for quantum theory. Yet modern quantum theory remains an extremely successful theory of matter that has given us many fascinating insights into the nature of matter and makes many quantitative predictions that have been borne out by experience. How is this possible?

F. Consider the Schrödinger cat thought experiment. According to the text book account of quantum measurement, immediately prior to our opening the box, the cat is in a 50-50 superposition of alive and dead states; when we open the box and look at the cat, we trigger a collapse into just one of those states. Most people find that instinctively implausible. However our instincts have mislead us often enough. We all felt instinctively that there is a universal fact over whether two events are simultaneous; or that the sum of the angles of a right angle has to be 180 degrees. Both proved to be false. Should we believe our instincts in this case? If so, why? If not, why not?