Zeno's paradoxes and supertasks

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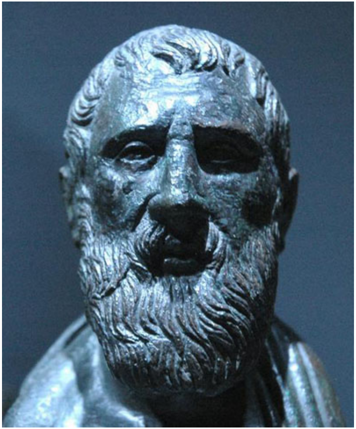
Zeno of Elea (ca. 490-430 BCE)



- Against the mathematical description of change, he launches attack against all conceptions of change we considered last time.
- What he tries to show is that those who ridiculed Parmenides for denying change are committed to views just as absurd.
- Specifically, they cannot offer a consistent mathematical description of the change they believe exists.

Some material here courtesy of John Earman's course on *Paradoxes*, University of Pittsburgh.

Zeno's paradox of the racetrack (or 'dichotomy')

- Atalanta, mythical hunter of greek antiquity, to run a 100m dash
- Zeno: how can an infinite number of (finite) runs, however small, ever be completed (in a finite amount of time)?

Zeno's paradox of the racetrack (suite)

(P1) If Atalanta wants to run a finite distance AB, she first has to run the first half of the distance.

(P2) Once she has run the first half of the distance, she must next run the first half of the second half from A to B.

⋮

(P3) Thus, Atalanta must run infinitely many (partial) distances in order to get to B.

(P4) It is logically (or physically?) impossible to run infinitely many distances—even for Atalanta.

(C) It is impossible to get from A to B.

Exercise

An 'inversion' of the argument claims that it is even impossible to start moving away from A. Can you reconstruct it?

Alternative formulation

(P1) If a task consists of a number of subtasks, then in order to complete the task, there must be a last subtask and that last subtask must be completed.

(P2) Atalanta's task—that of getting to the finish line—consists of an infinite number of subtasks: first covering one half of the distance, then covering one half of the remaining distance, etc. *ad infinitum*.

(P3) There is no last subtask for Atalanta.

(C) Therefore, Atalanta cannot complete the task.

Alternative formulation (suite)

Compare (P1) to

(P1') If a task consists of a number of subtasks, then in order to complete the task, all of the subtasks must be completed.

(P1') is surely true. But (P1') does not imply (P1). (P1) is true if the number of subtasks is finite (in which case (P1) and (P1') are equivalent); but (P1) is false if the number of subtasks if infinite—as is shown by Zeno's construction!
Alternative formulation of argument is valid, but not sound!

Analysis

Fact

An infinite series of finite numbers can have a finite sum.

- Mathematically: the distance is split into pieces:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 1 \qquad (1)$$

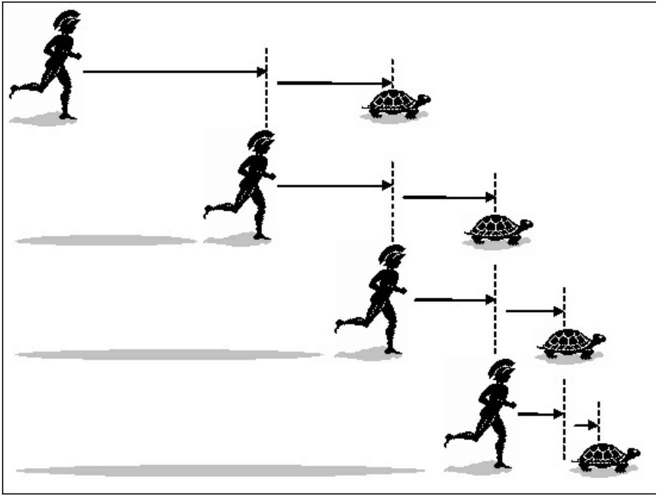
- The corresponding series of partial sums is $1/2, 3/4, 7/8, \ldots$. This series converges to 1.
- In other words, for an arbitrary $\epsilon > 0$, there exists an n such that for all $m \geq n$,

$$|1 - m\text{th partial sum}| < \epsilon. \qquad (2)$$

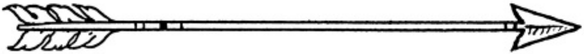
Analysis

- So mathematics can accommodate a suitable notion of infinite sums—but this notion only emerged with Augustin-Louis Cauchy (1789-1857).
- This finally vindicates Aristotle's response to Zeno that a (potential) infinity of intervals does not (need to) sum to infinity.

Similar: the paradox of Achilles and the Tortoise



The paradox of the arrow



Assumption

Time has smallest parts ('instants').

- This assumption works for continuous or discrete time.

The paradox of the arrow

(P1) At any instant the arrow is not moving.

(P2) Any stretch of time is composed of (nothing but) instants.

(C) Therefore, over any stretch of time the arrow does not move.

Various ways to understand (P1):

- In the duration of an instant, the arrow does not move. Then (P1) is true. But the argument is not valid.
- The arrow has zero velocity at each instant. Then the argument is valid (assuming that the motion of the arrow is continuous). But under this interpretation, (P1) is (or can be) false.

⇒ Either way, the argument is not sound—on one reading the argument is not valid, and on the other reading a premise is false.

The paradox of the arrow: another reconstruction

(1) A body is moving iff it changes its place in space. (Assum)

(2) At any instant of time a flying arrow is only in one place. (Assum)

(3) At any instant a flying arrow is not moving. (from (1) and (2))

(4) What is true for each instant of time is true for the whole time. (Assum)

(5) The flying arrow does not move. (from (3) and (4))

(5) is clearly self-contradictory. But there are at least two ways to solve the paradox:

The paradox of the arrow: another reconstruction

1. The argument contains the fallacy of composition (in (4)): that something is true for each component does not necessarily make it true for the whole.
2. There is something wrong about the way motion is being treated: whether something is moving or not cannot be decided by looking at the object at just one instant of time. What matters is how its place changes through different instants of time. This means that assumption (1) is incorrect. It should say

   (1') A body is moving iff it changes its place in space over time. (Assum)

   With (1) replaced by (1'), we can no longer proceed to (3) and the contradiction is avoided.

Supertasks: Zeno's revenge

Definition

Supertask A supertask is a task the fulfillment of which requires the fulfillment of infinitely many physically separate and distinct actions or operations in a finite amount of time.

Question

Is it logically and physically possible to fulfill such supertasks?

- Zeno's racetrack is not strictly speaking a supertask, as the actions are not separate and distinct.

Staccato run

A runner runs at an average speed of one mile an hour, but the run is broken up into shorter and shorter runs, at an average speed of two miles an hour, plus a rest of equal time:

Stage	Action	Time
1	run from start to 1/2 mile	0-1/4 hour
	pause for 1/4 hour	1/4-1/2 hour
2	run from 1/2 mile to 3/4 mile	1/2-5/8 hour
	pause for 1/8 hour	5/8-3/4 hour
3	run from 3/4 to 7/8 mile	3/4-13/16 hour
	pause for 1/16 hour	13/16-7/8 hour

etc.

- This is a genuine supertask since the actions are separate and distinct.

Thomson's lamp

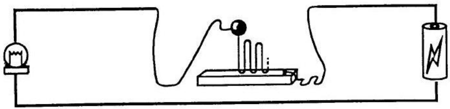
A lamp is switched on and off according to the following schedule:

Stage	Time	Action
1	1 min to midnight	switched ON
2	1/2 min to midnight	switched OFF
3	1/4 min to midnight	switched ON
4	1/8 min to midnight	switched OFF
etc.		

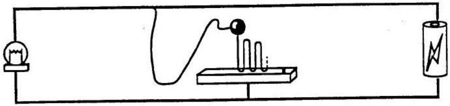
- Thomson claimed that it is logically impossible to complete this supertask. He argued as follows.
- At midnight, the lamp must be either ON or OFF. But either answer seems unacceptable.
- It cannot be ON, since every stage at which it was turned ON is followed by a stage at which it was turned OFF.
- And it cannot be turned OFF, since every stage at which it was turned OFF is followed by a stage at which it was turned ON.

Thomson's lamp: analysis

- Both solutions (ON and OFF) are logically consistent. It's just that not enough information is given to determine whether the lamp is ON or OFF at midnight.
- If we furnish further information, we see that both options can be realized by electric circuits (in the first circuit, the lamp stays ON at midnight, in the second, it remains OFF):



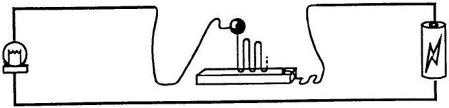
Contact switches lamp ON



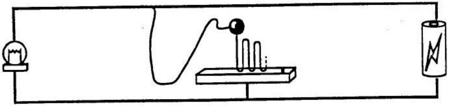
Contact switches lamp OFF

Thomson's lamp (suite)

Both solutions (ON and OFF) are logically consistent. Also possible are electric circuit which realize both alternatives:



Contact switches lamp ON



Contact switches lamp OFF

In the first circuit, the lamp stays ON at midnight, in the second, it remains OFF.

Bernadete's paradox

- Now an infinity of gods try to prevent Atalanta from running at all... (describe setup)
- However: it is not Atalanta's motion that is impossible, but instead the gods' plans—their superintention —is incompatible (although each seems capable of fulfilling their individual intention).
- Huggett: example of Arlo and Zowie (25)

Don't be too quick to dismiss Zeno's paradoxes...

Bertrand Russell (1903). *Principles of Mathematics*. Cambridge University Press, Section 325.

> *Having invented four arguments all immeasurably subtle and profound, the grossness of subsequent philosophers pronounced him to be a more ingenious juggler, and his arguments to be one and all sophisms. After two thousand years of continual refutation, these sophisms were reinstated, and made the foundation of a mathematical renaissance.*