Homework Assignment 7

25 Science, Philosophy, and the Big Questions due 2 March 2012

For submission

1. Name a spacetime that has the following properties:

   (a) It is uniformly filled with matter that is everywhere at rest.

   (b) It is empty of matter but space collapses and then expands everywhere.

   (c) It has a special center in the geometry of its space.

   (d) It has no matter and no gravitational effects anywhere.

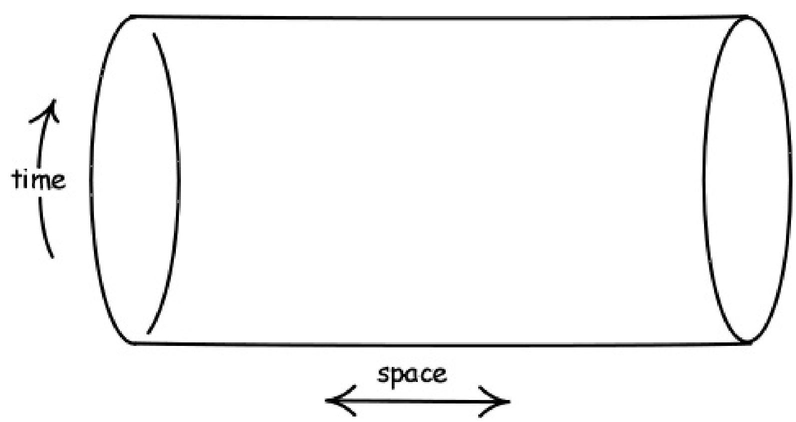
2. (a) What are Einstein's gravitational field equations of 1915? How does Einstein's cosmological constant λ modify them?

   (b) Show how the term can be re-interpreted as representing a form of matter in space.

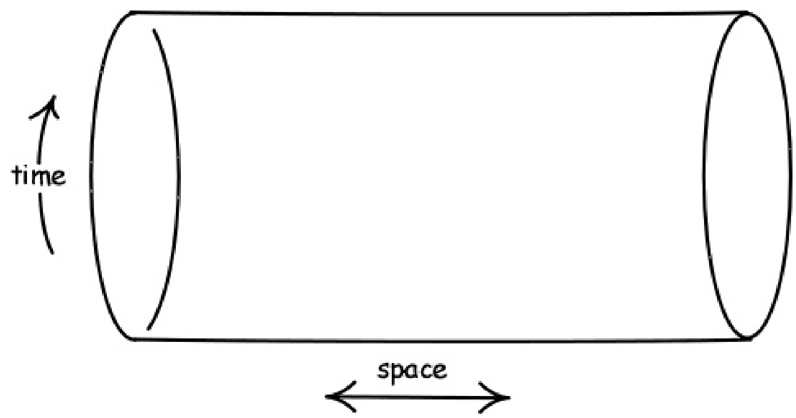
   (c) Why is the form of matter odd?

3. Imagine a time-travel, cylinder universe which is empty except for one mass.

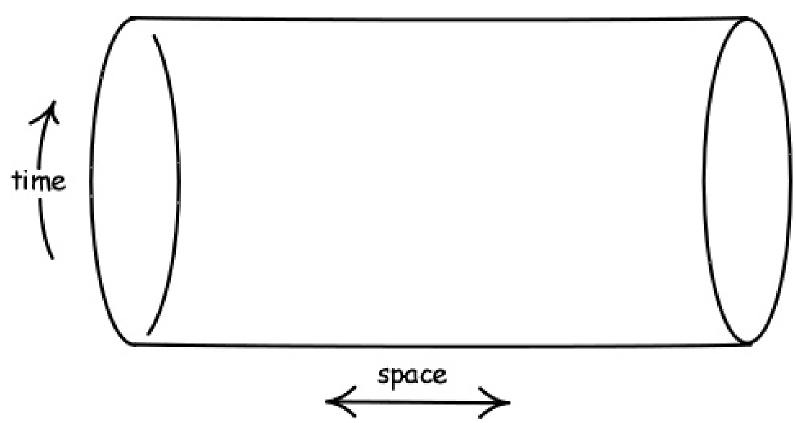
   (a) Draw in the worldline of the mass when it remains at rest in the space and reconnects with itself.



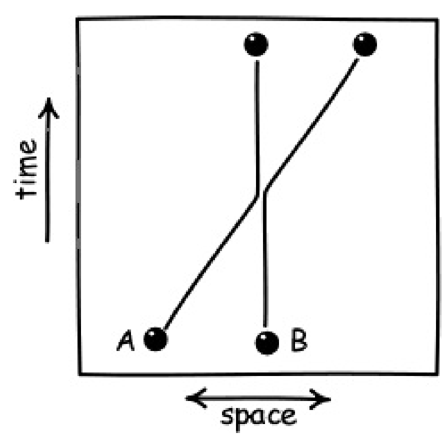
   (b) Draw the worldline of the mass when the mass moves to the right.



(c) The mass can collide with its future self. The collision is such that the mass gets deflected by just the right amount to come back as the later self of the collision. Draw the worldline that shows this, recalling that aside from collisions the mass moves inertially–i.e. in a straight line in the space.



Hint: Here's a way of resolving collisions in a spacetime diagram. The diagram below shows what happens in spacetime when a body A approaches a body B at rest and with equal mass. If the collision is elastic, body A comes to rest and body B moves off the same velocity that A had initially.



We can use Hubble's law to arrive at a crude estimate of the age of the universe. That is, we will calculate how long ago all the galaxies were crammed into our neighborhood of space. This time will be our estimate of how long ago the big bang happened. We will assume that each galaxy has moved at a constant speed for all time, although this speed will vary from galaxy to galaxy.

We will use the value of 20 km/sec per 1,000,000 light-years for Hubble's constant.

4. (a) If a galaxy is 1,000,000 light-years away from us now, according to Hubble's law, how fast is it receding from us?
   (b) A galaxy traveling at 1 km/sec will travel one light-year in 300,000 years. How long does the galaxy of (a) require to travel a light-year?
   (c) How long did it take the galaxy of (a) to get to its position 1,000,000 light-years distant from us?

5. Repeat the calculation of 4. for a galaxy now 2,000,000 light years distant from us.

6. Repeat the calculation of 4. for a galaxy now 3,000,000 light-years distant from us.

The final result of 4., 5., and 6. should be the same. At the time calculated, all the matter of universe would have been compressed into our neighborhood. This is our estimate of the age of the universe, often called the 'Hubble age'.

For discussion in the recitation

Some questions about relativistic cosmology:

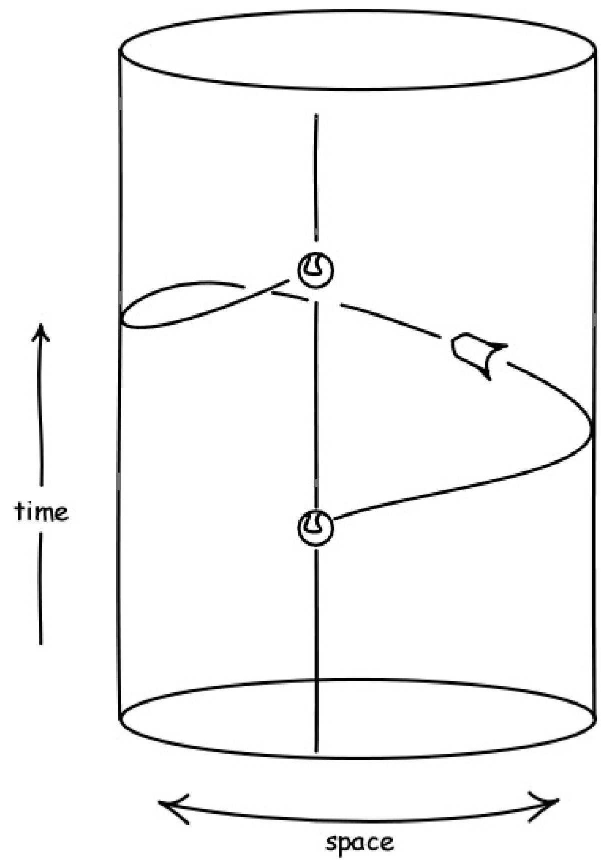
A. (a) If time travel were possible, the familiar paradox tells us that we could travel back in time, assassinate our grandfather in his youth, thereby precluding our birth. A contradiction ensues, since it now follows both that you traveled back in time and that you did not travel back in time. Good physical theories cannot tolerate contradictions. Does this mean we should abandon any theory that tells us that time travel is possible?

(b) In an old movie, a time traveler enters William Shakespeare's room just at the moment he is writing Hamlet's famous soliloquy. Shakespeare, however, is completely stumped and cannot find the right line. "To be or not to be." the time traveler whispers impatiently in Shakespeare's ear. "An excellent line", Shakespeare exclaims as he dutifully writes it in his manuscript. The puzzle is this: who thought up the line? More generally, is this the same sort of paradox as the 'grandfather paradox'? Or is there something significantly different about it.

(c) Here's another version of the paradox of (b). A time traveler steals Michelangelo's famous statue of David from its gallery in Florence and transports it back to Michelangelo's workshop in 1501, just as the sculptor is about to start work on the statue. The time traveler kidnaps the sculptor, keeps him trapped for the 3 years needed to sculpt the masterpiece and places the stolen statue in Michelangelo's workshop. When he is released, Michelangelo is too embarrassed to admit that he did not make the statue. Who made the statue?

B. Imagine a Minkowski spacetime wrapped up in one spatial direction. A space traveler synchronizes his clock with one on earth and then leaves earth. The traveler moves inertially eventually coming back to earth without ever changing direction. When the traveler's clock and the earth clock are compared, the traveler's clock will be found to have been slowed by the motion and will read less than the earth clock. Is this a violation of the principle of relativity? Shouldn't the traveler expect the earth clock to have run slower? Note that this version of the 'twin' problem is unlike the familiar one in so far as the traveler moves inertially at all times; there is no turning around and thus no acceleration.

(Hint: this space has a preferred state of motion! To find it, try drawing in the hypersurfaces of simultaneity of the earth and of the spacetraveler.)



Some questions about big bang cosmology:

C. It may seem that Hubble's law conflicts with the basic supposition of Friedman Robertson Walker cosmology that the universe is homogeneous and isotropic in space. For Hubble's law tells us that everything is rushing away uniformly from our particular galaxy. Does not that make our galaxy some sort of special center of galactic motion, different from every other galaxy? The following calculations show that the galactic motions of Hubble's law look the same from every galaxy.

Consider (0) our galaxy and galaxies (I) 1,000,000 and (II) 2,000,000, and (III) 3,000,000 and (IV) 4,000,000 light years distant from us, all in the same direction. Compute the velocities of recession of the galaxies (I)-(IV) from us.

Now imagine that you are an observer located on galaxy (I). Recompute the velocities of recession of the other galaxies. Find that Hubble's law still holds. That means that the expansion looks the same to an observer on galaxy 1 as it does from our galaxy. It is not hard to see that the same result will hold for all observers, no matter which galaxy is their home.

(In computing these velocities, use the ordinary Newtonian rule for composing velocities.)

D. If the universe turns out to have an open geometry so that space is infinite, then all of our observations are showing us only the tiniest part of space. It is a finite fragment of an infinite expanse. Given that tiny sample, are we justified in asserting that the universe is spatially homogeneous–the same in every place? Or is this fundamental hypothesis of cosmology mere supposition?

E. Some theorists find a singularity, such as the big bang, an affront to science and feel a strong need to find reformulated theories that will eliminate them. Are singularities to be avoided or eliminated from theories if possible? Why?

F. The adoption of big bang cosmology triggered a long standing debate in theology. Should we take the big bang to vindicate the theistic claim of divine creation of the universe? Theists like to point out the similarity between the creation account in Genesis–"Let there be light."–and big bang cosmology's assertion of a finite past that was dominated by radiation as we approach the big bang. Atheists, however, reply that nowhere in big bang cosmology do we find any agent outside of space, time and matter with creative powers; we just have matter and space expanding in time.