Topic 10: Bohm's Theory

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146 Philosophy of Physics: Quantum Mechanics

David Bohm (1917-1992)

- Berkeley PhD 1943
- Princeton, U of Sao Paolo, Technion Haifa, U Bristol, Birkbeck College London
- In 1949, the House Un-American Activities Committee called upon Bohm to testify before it, but he pleaded the 5th amendment (right to decline to testify) because he didn't want to give evidence against any of his colleagues.
- was arrested in 1950 for non-cooperation with the Committee, but was acquitted in 1951
- Princeton refused to re-instate him, despite the pleads by Einstein

Bohm's theory in a nutshell

Albert, Ch. 7.

- almost exactly the same empirical content as standard QM
- much the same mathematical apparatus
- different metaphysics: "every material particle in the world invariably has a perfectly determinate position." (Albert, 134)
- deterministic dynamics
- dynamics only appear to be probabilistic, because of our epistemic limitations

⇒ gives probabilities the same role as in classical physics

- wave functions are thought to be physical entities (although distinct from particles)—and not like a mathematical representation of a system's state

- wave functions are physical entities whose properties are their amplitudes at every given point in space
- dynamical evolution is completely governed by the linear Schrödinger equation, without nonlinear episodes of collapse
- wave functions ‘push the particles around’
- additional laws in theory how this pushing, this ‘guidance’ happens
- But wait a minute: didn’t von Neumann (1932) and Bell (1964) prove that there can’t be a HV-theory??

Von Neumann's classic argument, Bell's theorem

- von Neumann (1932, 325 of English tr.) argued that no HV-theory could possibly make the same statistical predictions as QM
- Bell (according to Mermin 1993, 805): bogus, Neumann's assumptions about what a HV-theory is committed to are unreasonably strong, so much so that Bell exclaimed that "the proof of von Neumann is not merely false but foolish!"
- OK, so von Neumann killed a strawman, but didn't Bell (1964) rule out HV-theories?
- Wigner (1976): "In my opinion, the most convincing argument against the theory of hidden variables was presented by J.S. Bell (1964)."
- Except that... it wasn't an argument against HV-theories!
- Bell's theorem, in fact, only shows that QM is irreducibly nonlocal (but you knew that, didn't you?).

Setting things up

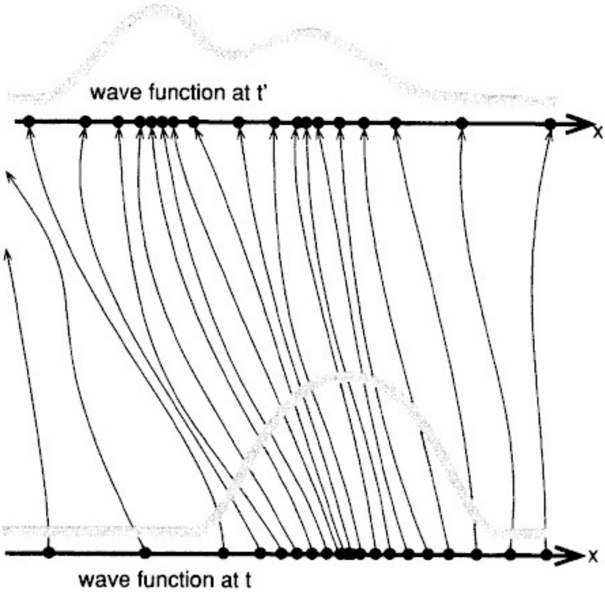
- velocity of a particle given by the value of the velocity function $V(x)$ of the position x
- this function is to be evaluated at point P of where particle is located at the time
- $V(x)$ for any particle at any time is given by the particle's wave function at that time by means of an algorithm $V[...]$
- For a structureless particle in one dimension, the velocity function at a particular time is equal to function $V[\psi(x)]$ where $\psi(x)$ is the particle's wave function at the same time.

$\Rightarrow$ "position of a particle and its wave function at any given time can in principle be calculated with certainty... from its position and its wave function at any other time, given the external forces to which that particle is subject in the interval between those times." (137)

The swarm metaphor

Imagine a gigantic swarm of particles, distributed at the initial moment t, in perfect accordance with $\psi(x)$, i.e. the density of particles along the x-axis is everywhere equal to the square of the absolute value of the wave function at t, $\psi(t, x)$.

> *"If that's so (and this is the punch line), then it can be shown to follow from the dynamical equations of motion for the wave function and from the form of the algorithm for the calculation of the velocity functions that the density in space of the positions in the later swarm (no matter what later time t' happens to be) will likewise be everywhere equal to the square of the absolute value of the wave function then (at the later time)... What happens... is that the particle gets carried along with the flows of the quantum-mechanical probability amplitude in the wave function, just like... a cork floating on a river." (138f)*


wave function at t'
x
x
wave function at t

The statistical postulate

A fundamental postulate of Bohm's theory:

Postulate (Statistical Postulate)

"[I]f you're given the present wave function of a certain particle, and if you're given no information whatever about the position of that particle, then... what ought to be supposed... is... that the probability that the particle is presently located at any particular point in space is equal to the square of the absolute value of its present wave function at that point." (140)

⇒ Postulate guarantees that Bohm's theory entails precisely what Principles C and D entail about the probabilities of finding particles at any particular location at any particular future time

Extension to more than one DOF

- Extension to three spatial dimensions is straightforward: the wave function takes on values in three dimensions, and we need three algorithms to compute velocities in all spatial dims
- Extension to include spin properties: spin props, on this view, are mathematical properties of the wave functions which play no role in determining the velocities (velocities are determined from the coordinate-space part of the wave function alone):

$$V_i(x, y, z) = V_i[|\text{e.g. black}\rangle|\psi(x, y, z)\rangle] = V_i[\psi(x, y, z)] \tag{1}$$

- If the system is in a state that is nonseparable between the spin and coordinate DOFs, e.g.

$$\alpha|\text{black}\rangle|\psi(x, y, z)\rangle + \beta|\text{white}\rangle|\phi(x, y, z)\rangle, \tag{2}$$

then each of the different CS-wave function parts contribute, in accordance to their 'weight', to determining the velocity functions:

$$V_i(x,y,z) = \frac{|\alpha\psi(x,y,z)|^2 V_i[\psi(x,y,z)] + |\beta\phi(x,y,z)|^2 V_i[\phi(x,y,z)]}{|\alpha\psi(x,y,z)|^2 + |\beta\phi(x,y,z)|^2}$$

- Extension to multipartite systems is a bit more involved. Let's do this step by step.
- Suppose $|\psi_{1,2}\rangle$ is arbitrary quantum state of a bipartite system, $|\hat{X}_1 = x_1, \hat{X}_2 = x_2\rangle$ represents the state of the system when the first particle is located at x_1 (in 3-d space) and the second is at x_2.

$\Rightarrow$ Two-particle wave function associated with state $|\psi_{1,2}\rangle$ is given by a function of x_1 and x_2:

$$\psi(x_1, x_2) = \langle\psi_{1,2}|\hat{X}_1 = x_1, \hat{X}_2 = x_2\rangle.$$

- Bipartite wave function can be regarded as a function of position in (somewhat more abstract) 6-dim space, first three referring to position of particle 1 and second three to that of particle 2

$\Rightarrow$ a point in this 6-dim space represents particular positions for both particles

- "The laws of Bohm's theory for two-particle systems which stipulate precisely how the two-particle wave functions push such pairs of particles around are formulated just as if it were a single particle that were being pushed around in a six-dimensional space." (143)
- Velocities given by algorithm from velocity functions of position of two-particle system in 6-d space:

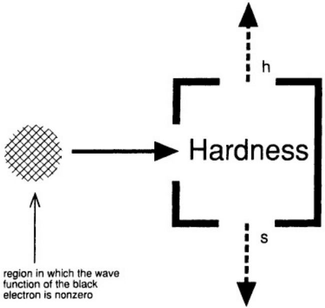
$$V_i(x_1, y_1, z_1, x_2, y_2, z_2) = V_i[\psi(x_1, y_1, z_1, x_2, y_2, z_2)],$$

where $i = x_1, y_1, z_1, x_2, y_2, z_2$.

- The statistical postulate gets generalized in a straightforward way, such as to ascertain that Bohm's theory entails exactly the same predictions as the two-particle versions of Postulates C and D of ordinary QM entail about the probabilities of finding the particles in particular locations in 3-d space.
- This "can be construed as stipulating something about the initial conditions of the universe;... as stipulating that what God did when the universe was created was first to choose a wave function for it and sprinkle all of the particles into space in accordance with the quantum-mechanical probabilities, and then to leave everything alone, forever after, to evolve deterministically." (144f)

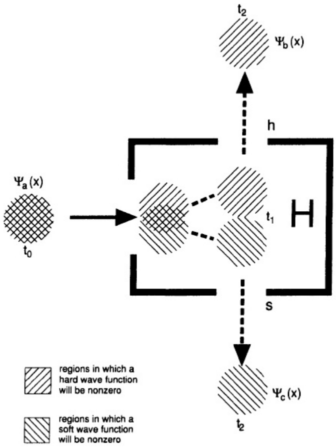
Application to concrete measurement situations

- look at measurements with spin boxes
- Main point: "all of the future positions of this electron... can in principle be determined, with certainty, from its present position" (145) (including the aperture from which the e^- exits the spin box)



$$|\text{black}\rangle|\psi_a(x)\rangle \longrightarrow \frac{1}{\sqrt{2}}(|\text{hard}\rangle|\psi_b(x)\rangle + |\text{soft}\rangle|\psi_c(x)\rangle) \tag{3}$$

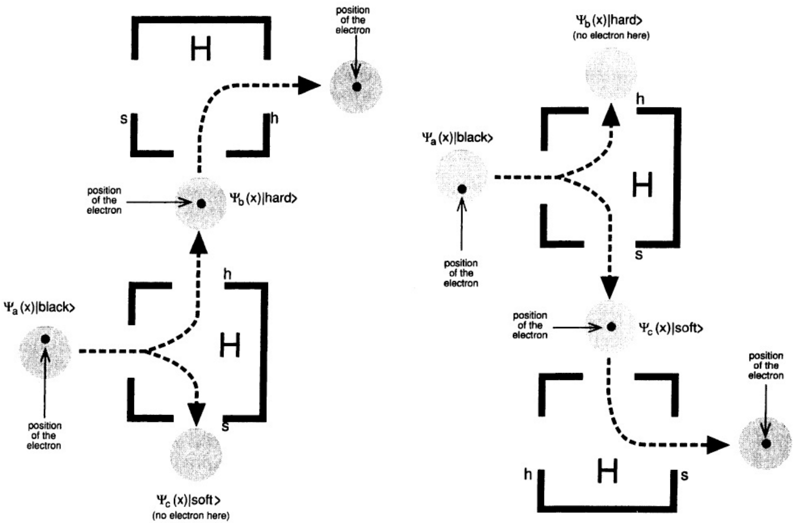
(indices to wave functions indicate the region where they are nonzero)



Wherever the particle exactly happens to be, it'll be carried along by the 'local currents' of the quantum-mechanical probability amplitudes. This entail that, without loss of generality,

> *"in the event that the electron starts out in the upper half of the region where $\psi_a(x)$ is nonzero, then it will ultimately emerge from the hard aperture of the box; and in the event that the electron starts out in the lower half of that initial region, then it will ultimately emerge from the soft aperture of the box." (147)*

If the e^- emerges from the hard (soft) aperture, and is subsequently fed into another hardness box (without permitting the hard and soft branches of the wave function to be reunited), it will, with certainty, emerge from the hard (soft) aperture of the second box because in between boxes, it moves through regions of space in which the amplitude of the soft (hard) part of the wave function is zero: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Ψb (x)|hard>
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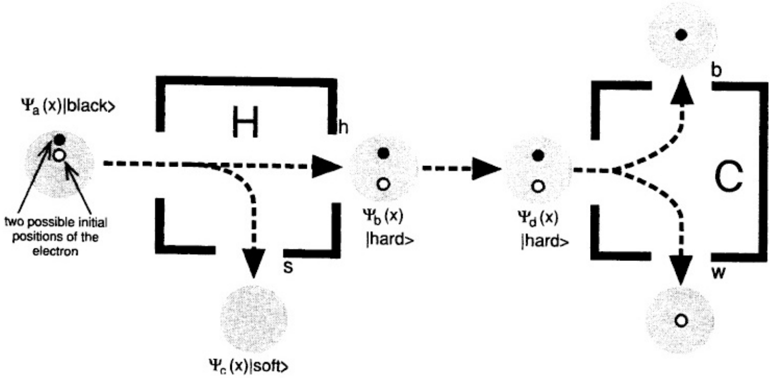
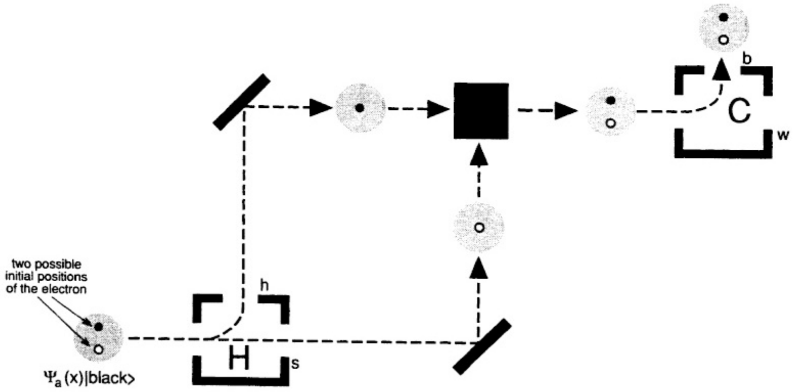
What happens here?



Two-path experiments

If no walls are inserted along the way:



A subtlety

- If, however, a wall is inserted along one of the paths, then certain initial positions of the e^- are such that if the e^- was initially there, it will hit the wall and not reach the box of reunification.
- In this case, certain other initial positions will entail that the e^- will finally emerge through the white aperture (and some initial positions will entail that it eventually emerges through the black aperture).
- But that's curious: when there was no wall, all initial positions in region *a* entailed that the e^- would finally emerge from the black aperture, but that's no longer the case for all initial positions in *a* in the presence of a wall!

Another curiosity: contextuality

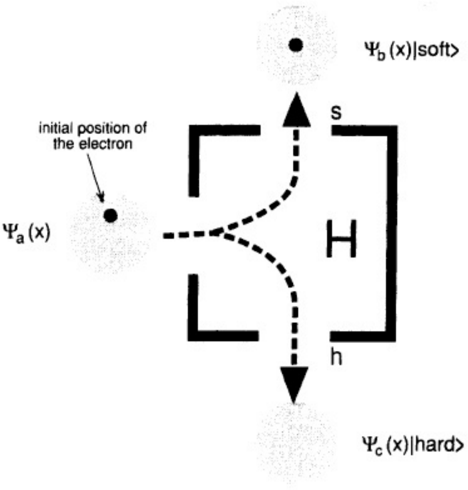
- In the original set-up, we had

$$|\text{black}\rangle|\psi_a(x)\rangle \longrightarrow \frac{1}{\sqrt{2}}(|\text{hard}\rangle|\psi_b(x)\rangle + |\text{soft}\rangle|\psi_c(x)\rangle).$$

- But if we rotate the hardness box such as to change its orientation (as on the next slide), the e^- evolves according to

$$|\text{black}\rangle|\psi_a(x)\rangle \longrightarrow \frac{1}{\sqrt{2}}(|\text{soft}\rangle|\psi_b(x)\rangle + |\text{hard}\rangle|\psi_c(x)\rangle).$$

- The path of the particle through the box is exactly the same—as if the box hadn't been flipped!
- In other words, if the e^- is initially in the upper (lower) half of the region where $\psi_a(x)$ doesn't vanish, it now registers as soft (hard), just opposite of when the box wasn't flipped.


Ψb (x)|soft>
S
initial position of
the electron
Ψa (x)
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Ψc (x)|hard>

"And so (even though this is a completely deterministic theory) the outcome of this sort of 'measurement' of the hardness of an electron will in general not be pinned down in this theory even by means of a complete specification of the electron's position and its wave function, which is (after all) everything there is to be specified about that electron. The outcome of such a hardness measurement is in general going to depend, on this theory, on precisely how and under precisely what circumstances the hardness gets measured, even down to the orientation of the hardness box in space." (154)

"And so it doesn't quite make sense, in general, on this theory, to think of hardness as an intrinsic property of electrons or of their wave functions... Properties like that... have come to be referred to in the literature... as contextual. And it turns out that... every one of the traditional quantum-mechanical observables of particles other than position are contextual properties on this theory too... [T]here are theorems... to the effect that any deterministic replacement for quantum mechanics whatever will invariably have to treat certain such observables as contextual ones." (155)

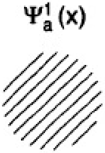
Nota bene: Properties associated with an eigenvector are not had by the particle contextually when it is in this eigenstate.

(Theorems: Gleason 1957, Kochen and Specker 1967)

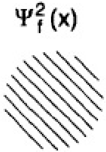
Nonlocality in Bohm's theory

- Nonlocality: in general, each of the six velocity functions will depend on entire bipartite system's position in 6-d space (if state is nonseparable between particles)
- Consider an (nonseparable) EPR-pair of e^- (coordinate-space part of wave functions separable, spin part nonseparable):

$$\frac{1}{\sqrt{2}}\left(|\text{hard}\rangle_1|\psi_a(x)\rangle_1|\text{soft}\rangle_2|\psi_f(x)\rangle_2 - |\text{soft}\rangle_1|\psi_a(x)\rangle_1|\text{hard}\rangle_2|\psi_f(x)\rangle_2\right)$$

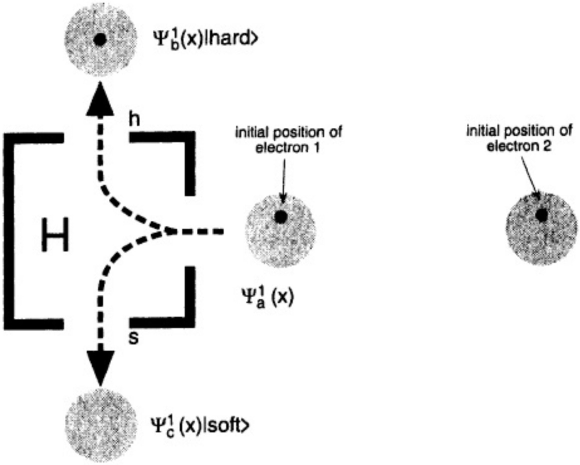


The coordinate-space wave function of electron 1



The coordinate-space wave function of electron 2

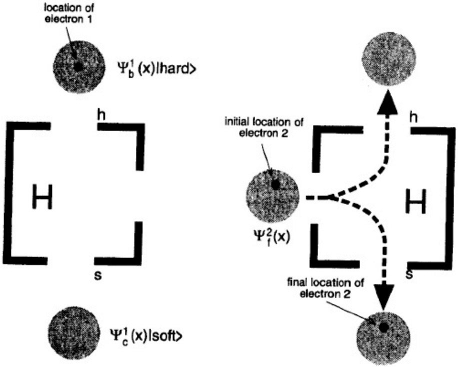
Suppose the first e^- passes through (right-side-up) hardness box:



$$\frac{1}{\sqrt{2}}(|\text{hard}\rangle_1|\psi_b(x)\rangle_1|\text{soft}\rangle_2|\psi_f(x)\rangle_2 - |\text{soft}\rangle_1|\psi_c(x)\rangle_1|\text{hard}\rangle_2|\psi_f(x)\rangle_2)$$

- Since e$^-$ was in upper half of $\psi_a^1(x)$ and thus emerged through the hard aperture of the left box, the second term in the foregoing superposition is zero.

$\Rightarrow$ second branch has no effect on motion of either e$^-$, i.e. "electron 2 will behave, under all circumstances, as if its wave function were purely soft." (157)



Concrete nonlocality

Concrete form of nonlocality, action-at-a-distance:

> *"even though the wave function evolves here entirely in accordance with the linear dynamical equation of motion, the passage of electron 1 through the hardness box brings about... an effective collapse of the wave function of the entire two-electron system, instantaneously, no matter how far apart they may happen to be or what may happen to lie between them." (158)*

The tension with Special Relativity (SR)

- If we're given the positions of both EPR particles AND the wave function of the bipartite system, then we could instantaneously transmit discernible information over vast distances. (Why?)

$\Rightarrow$ violation of SR (which prohibits superluminal signalling)

- This instantaneous signalling would require a preferred frame of reference, or, in other words, absolute simultaneity.
- But if we're only given the wave function of the bipartite system (which is all we CAN be given, anything else can be shown to violate QM; cf. 164-9), then instantaneous signalling is impossible.
- Also, it'll be impossible to determine which frame of reference is in fact the preferred one.
- "[T]aking Bohm's theory seriously will entail being instrumentalist about special relativity." (161)

Real collapse vs. effective collapse

"[T]he business of... empirically confirming that as a matter of fact [the determinateness e.g. of the hardness of an electron] is merely effective, and that nothing has actually collapsed, will be... quite fantastically difficult... Bohm's theory is going to make things look (for all practical purposes) as if wave functions do collapse when we do measurements with instruments with macroscopic pointers... even though it can in principle be empirically confirmed (on this theory) that in fact they don't occur at all... the statistical postulate will... entail that the epistemic probabilities of the various possible effective collapses... will all be precisely the same as the ontic probabilities of the corresponding actual collapses... in quantum mechanics." (163f)

In conclusion

“This is the kind of theory whereby you can tell an absolutely low-brow story about the world... in which the whole universe always evolves deterministically and which recounts the unfolding of a perverse and gigantic conspiracy to make the world appear to be quantum-mechanical.

"And that conspiracy works... like this: Bohm's theory entails everything that quantum mechanics entails... about the outcomes of measurements of the positions of particles in isolated microscopic physical systems; and moreover it entails that whenever we carry out a measurement of any quantum-mechanical observable whatever, then... the measured system... will subsequently evolve just as if that system's wave function has been collapsed, by the measurement, onto an eigenfunction... of the measured observable, even though as a matter of fact is hasn't been; and it also entails that the probabilities of those 'collapses' will be precisely the familiar quantum-mechanical ones." (169f)