From Plato to Mill

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124 Philosophy of Mathematics

Plato ($\Pi\lambda\acute{\alpha}\tau\omega\nu$, 428/7-348/7 BCE)

- "The safest general characterisation of the European philosophical tradition is that it consists of a series of footnotes to Plato." (Alfred North Whitehead, *Process and Reality*, 1929).
- framed many philosophical issues in epistemology, metaphysics, political philosophy, ethics
- philosophical work in forms of dialogues (early, middle, late)
- *Meno* is usually considered a transitional dialogue between the early and middle periods
- *Theaetetus*: what is knowledge? True judgment with an account (although even that is ultimately unsatisfactory)

Plato and horses

Plato's Theory of Forms and the World of Being

- $\exists$ realm of abstract objects called 'Forms'
- these Forms exist independently from human mind; they are unchanging, eternal, perfect
- explain what multiple concrete, particular things have in common ('universals')
- realm of Forms constitutes reality and is more perfect than dim reflection of it that humans experience and thus enables us to have concepts of perfect things (e.g. perfect circle)
- What we know as red is only afterimage or a corporeal display of the Form of Redness.
- Examples of virtues and bees: Socrates: "even if [the Xs] are many and various, they must still all have one and the same form which makes them [X]." (*Meno*, from M. Huemer (2002), 132)
- How do we come to know or apprehend these Forms?

Michael Huemer (ed.), *Epistemology: Contemporary Readings*, Routledge (2002).

The 'paradox of knowledge'

"Meno: And how are you going to search for [the nature of virtue] when you don't know at all what it is, Socrates? Which of all the things you don't know will you set up as target for your search? And even if you actually come across it, how will you know that it is that thing which you don't know?" (in Huemer, 134)

Characterization (Paradox of knowledge)

Either you do or do not know something particular. If you don't know it, then how could you possibly recognize it when you see it? If you do know it, then you don't need to look for it. So why should we bother attempting to gain knowledge?

Meno: Innate knowledge

- solution in the *Meno*: people have innate knowledge that they can recall
- exemplified in slave boy who 'knows' geometrical principles, but must be helped in recovering them from memory
- Theory of Anamnesis (recollection): soul is immortal, being constantly reincarnated, knowledge is forgotten in shock of birth, learning is bringing back or recollecting this hidden knowledge in ourselves
- Socrates is not really teacher, but 'midwife' aiding rebirth of knowledge

Question

How could an empiricist react to this account? How could a rationalist who doesn't believe in innate knowledge give an alternative explanation of the slave boy's 'learning'?

Plato's analogy of the divided line

	METAPHYSICS: OBJECTS	EPISTEMOLOGY: STATES OF MIND	
INTELLIGIBLE WORLD	higher forms (the Good is highest)	understanding (*noesis*, direct intuition)	KNOWLEDGE (*episteme*)
	mathematical forms	reason (*dianoia*, discursive reasoning)	
VISIBLE WORLD	sensible particulars	perception (*pistis*, common sense belief)	OPINION (*doxa*)
	images	imagination (*eikasia*, mere imagining)	

Plato on mathematics: double realism

- Plato: propositions of geometry and arithmetic are objectively true or false

⇒ realist in truth-value

- subject-matter of geometry is realm of geometric objects, which exist independently of mind, language, etc

⇒ realism in ontology

- geometric knowledge a priori, independent of sensory experience
- main reason: Platonist ontology (geometry not about physical objects in physical space), which cannot be apprehended by perception by only by understanding or reason
- Plato on numbers: numbers are either ratios of geometric magnitudes or collections of pure units (but there's no consensus on how to read Plato here)

Mathematics on Plato

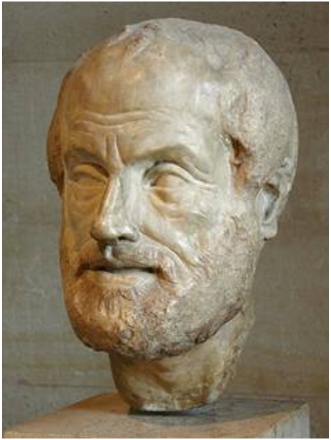
- Thinking about mathematics also shaped Plato's philosophy.
- He rejected the Socratic method of weeding out false beliefs and confusions because it never results in certainty.
- Instead, philosophy ought to model itself on geometry and its method of demonstration in attempt to obtain certain knowledge.

⇒ mathematical knowledge becomes paradigm for all knowledge, including metaphysics and moral knowledge.

- In fact, he proposed that all people trained as philosophers should study mathematics for ten years—more than we require today for professional mathematicians!
- and let no one ignorant of geometry enter his Academy!

⇒ "for Plato the fumbling but exciting and egalitarian Socratic method first gives way to the elite rigour of Greek mathematical demonstration" (63; and then to "an even more elite 'dialectical' encounter with the Forms")

Aristotle (’Αριστοτέλης) (384-322 BCE)



- student of Plato, teacher of Alexander
- systematized and developed knowledge from logic to metaphysics to physics to meterology, zoology, biology, poetry, drama, music, rhetoric, linguistics, politics, ethics, mathematics and more
- opposed Platonic rationalism, replaced it by empiricism
- Not a mere footnote to Plato!

Aristotle's philosophy of mathematics

- Aristotle rejects Plato's separate world of Being
- things have their Forms in the physical world; Forms inhere in sensible particulars
- (theme will recur when we consider structuralism *ante rem* vs. *in re*)

⇒ Aristotle is concerned with the nature of mathematical objects—he takes for granted that they exist, but the how thus becomes relevant to him

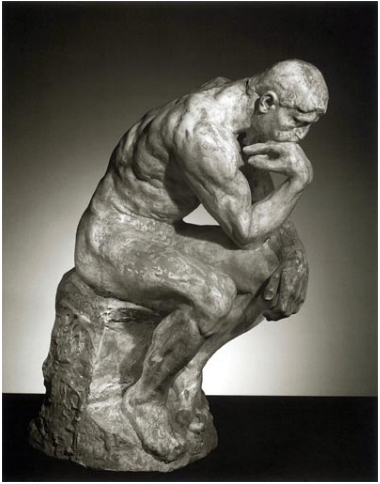
- physical objects 'contain' mathematical objects like lines, shapes, etc
- Of course, when mathematicians ponder mathematical objects, they "ignore certain physical aspects of their subject-matter" (66)
- But how does this 'ignoring' work?

Two (empiricist) interpretations of Aristotle

1. Abstraction: humans have faculty of 'abstraction', which lets them reflect on physical objects so as to abstract away from their particular physical features, thus 'creating' mathematical objects, or peeling them from their material garb
   - ⇒ mathematical objects don't exist prior to, or independent of, the physical objects they are abstracted from
   - double realism in truth-value and in ontology
   - Frege's challenge: abstraction undermines distinction, but "[w]hoever cannot distinguish between the things he is supposed to count, cannot count them either." (1971, 125)

2. Fictionalism: mathematical objects are merely useful fictions, even though mathematical statements are true or false of these fictional entities
   - maintain realism in truth-value, but give up realism in ontology
   - potential problem with mismatch between physical objects and mathematical ones (actually for both interpretations): how to explain that mathematical theorems are false of the actually existing imperfect physical objects
   - In response, Aristotle could claim that there actually are physical objects lacking these imperfections or make a move to modality.

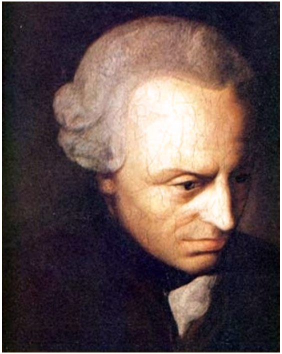
Reorientation



Rationalism and empiricism on mathematics

- common ground: mathematics a priori and necessary, is about physical magnitudes or extended objects
- differed over mind's access to ideas of these magnitudes/extended objects: rationalism directly through reason, empiricism derived from experience
- rationalism easily accounts for 'mismatch' between objects of senses and their mathematical analogues and for necessity, but struggles with 'match' between observed physical objects and their mathematical counterparts
- for empiricism, it is just the other way around, since mathematics indirectly studies relations among things we observe

Immanuel Kant (1724-1804): transcendental idealism



- one of the most influential thinkers of modernity, influence on both analytic as well as continental philosophy
- 1770: appointed Professor of Logic and Metaphysics at U of Königsberg, and briefly thereafter was awaken from his 'dogmatic slumber' by Hume
- 1770-1781: silent decade
- 1781/7: *Critique of Pure Reason*
- influential work in metaphysics, epistemology, ethics, aesthetics, political philosophy, philosophy of religion

Critique of Pure Reason (1781/87)

- rational human agent at center of cognitive activity
- synthesis of rationalist and empiricist positions
- rational order of world cannot simply be accounted for by sense perceptions
- conceptual unification and integration by active mind using ‘precepts’ (space, time) and following ‘categories of understanding’ (cause, substance) operating on manifold of sense perceptions
- consequently, objective causal structure of world depends upon mind
- mind makes ineliminable constitutive contribution to knowledge

Synthesis of rationalism and empiricism

"There can be no doubt that all our knowledge begins with experience. (B 1)... Thoughts without content are empty, intuitions without concepts are blind. (B 75) [Sometimes paraphrased as 'Concepts without percepts are empty, percepts without concepts are blind.']... Thus all human knowledge begins with intuitions, proceeds from thence to concepts, and ends with ideas. (B 730)" (*Critique*)

Kant's synthetic *a priori*

- *a priori* principles indispensable for possibility of experience (in order to endow experience with certainty)
- *a priori* knowledge delivers 'precepts' (space, time) and 'categories of understanding' (cause, substance) which operate (unify, integrate etc) on manifold of sense impressions and make experience possible
- all our *a priori* speculative knowledge must ultimately rest on synthetic/ampliative statements

$\Rightarrow$ metaphysics contains synthetic *a priori* judgments (e.g. 'Everything which happens has a cause.')

- natural science (physics) contains synthetic *a priori* judgments as principles (e.g. conservation principles)

The model character of mathematics

- model of how far *a priori* knowledge can be extended beyond scope of experience: mathematics

$\Rightarrow$ mathematical judgments are mostly synthetic *a priori*

- arithmetic: '$7 + 5 = 12$' is synthetic because concepts of '7' and '5' and of 'addition' do not contain concept of '12'; thus, conceptual analysis alone does not determine that $7 + 5 = 12$; '12' is constructed by 'pure' intuition
- analytic geometry: 'straight line between points is shortest' synthetic because 'straight' doesn't contain quantitative information
- Kantian epistemology: synthetic propositions knowable only through 'intuition'

The two features of Kantian intuition
(1) Singularity

- first feature: intuitions are singular, i.e. they concern individual objects, not general truths (contra conceptual analysis)

⇒ we cannot learn existential matters by conceptual analysis

- mathematics deals with individual objects, numbers, sets, geometric objects, etc
- Kant took even space itself to be singular and apprehended by intuition

The two features of Kantian intuition

(2) Immediacy

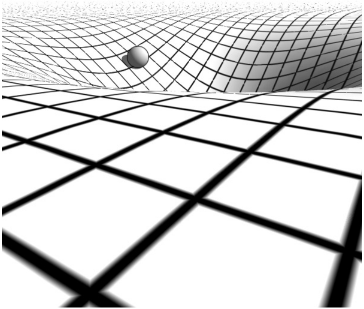
- second feature: intuition yields immediate knowledge
- 'pure' intuition gives "forms of possible empirical intuitions" (81), concerns the" forms of possible human perception" (ibid)
- pure intuition is "awareness of the spatio-temporal form of ordinary sense perception" (ibid)

⇒ mathematics (arithmetic and geometry in particular) give account of framework of perception

intuition	⇒	sense perception
'pure' intuition	⇒	forms of sense perception

Philosophical cognition is rational cognition from concepts. Mathematical cognition is rational cognition from the construction of concepts... [P]hilosophical cognition contemplates the particular only in the universal. Mathematical cognition, on the other hand, contemplates the universal in the... individual; yet it does so nevertheless a priori and by means of reason. (B741f) Philosophy keeps to universal concepts only. Mathematics can accomplish nothing with the mere concept but hastens at once to intuition, in which it contemplates the concept in concreto, but yet not empirically... (B743)

Problems for Kant's philosophy of mathematics

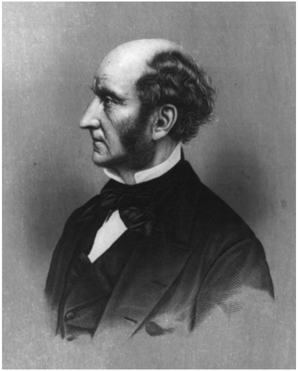


- Kant: parallel postulate is an a priori, (intuitively, though not conceptually) necessary truth

⇒ Kant: "we know a priori that non-Euclidean geometry cannot be applied in physics." (90)

- C19: non-Euclidean geometry, which gets applied to physical space in Einstein's general relativity (1915)

John Stuart Mill (1806-1873)



- philosopher, political economist, civil servant/MP
- precocious childhood
- philosophy of science, confirmation theory: *A System of Logic* (1843, principles of induction, covering-law model of explanation, best system theory of laws)
- ethics: *Utilitarianism* (1863)
- political philosophy: *On Liberty* (1859, harm principle), *The Subjection of Women* (1869, equality, women's suffrage)

Mill as post-Kantian philosophy of mathematics

- Given its problems, can we account for necessity and a priori nature of mathematics without invoking Kantian intuition?
- Empiricism: either all mathematics is analytic, or it is a posteriori, like Mill
- Mill: no significant knowledge about the world can be a priori
- But the propositions of mathematics are real knowledge about the world.

⇒ mathematics is synthetic a posteriori, appearance of necessity arises only from early and constant experience

- Mill's fundamental epistemological inference: enumerative induction

⇒ laws of mathematics can be traced to enumerative induction

Mill's philosophy of geometry

- mathematical propositions summarize experience, are generalizations (and don't really add to knowledge, which is about particulars)
- geometric objects = approximations of actual drawn figures, 'limit concepts'

⇒ geometry about idealizations of possibilities of construction

- "[T]he propositions of geometry are inductive generalizations about possible physical figures in physical space. They have been confirmed by long-standing experience." (94)
- problem with this: notion of possibility unclear (e.g., how is it 'possible' to draw a line between two points or bisect a line segment?); can't really be physical possibility

Mill's philosophy of arithmetic

- For Mill, numbers are numbers or ordinary things, ranging not over individuals, but over aggregates of objects.
- sums are real (as opposed to 'verbal') propositions about physical (as opposed to abstract) aggregates and their structure
- Some challenges for Mill (some coming from Frege):
    - make sense of 'collecting' and 'separating' (Maddy may help here: difference between seeing four shoes and seeing two pairs of shoes)
    - large numbers: how can we have experience of large aggregates?
    - Mill thinks each numeral represents size of actual collections of actual things, which means there are (or at least, could be) infinitely many things
    - Again, what is 'possible' experience? And how can we make sense of mathematical induction?
- Question: what about other branches of mathematics? Mill didn't have to say much...