

# Schematic inference and replacement rules

Greek letters symbolize statements, atomic or composite.

Phil 10

## 1.1 Modus ponens (MP)

$$\begin{array}{l} \phi \supset \psi \\ \phi \\ \hline \therefore \psi \end{array}$$

## 1.2 Modus tollens (MT)

$$\begin{array}{l} \phi \supset \psi \\ \sim \psi \\ \hline \therefore \sim \phi \end{array}$$

## 1.3 Disjunctive syllogism (DS)

$$\begin{array}{l} \phi \vee \psi \\ \sim \phi \\ \hline \therefore \psi \end{array}$$

## 1.4 Hypothetical syllogism (HS)

$$\begin{array}{l} \phi \supset \psi \\ \psi \supset \xi \\ \hline \therefore \phi \supset \xi \end{array}$$

## 1.5 Simplification (simp)

$$\begin{array}{l} \phi \bullet \psi \\ \hline \therefore \phi \end{array}$$

## 1.6 Dilemma (dil)

$$\begin{array}{l} \phi \supset \psi \\ \xi \supset \zeta \\ \phi \vee \xi \\ \hline \therefore \psi \vee \zeta \end{array}$$

## 1.7 Conjunction (conj)

$$\begin{array}{l} \phi \\ \psi \\ \hline \therefore \phi \bullet \psi \end{array}$$

## 1.8 Disjunction introduction (DI)

$$\begin{array}{l} \phi \\ \hline \therefore \phi \vee \psi \end{array}$$

## 2.1 Conditional exchange (CE)

$$(\phi \supset \psi) :: (\sim \phi \vee \psi)$$

## 2.2 Double negation (DN)

$$\phi :: \sim \sim \phi$$

## 2.3 Commutation (Comm)

$$\begin{array}{l} (\phi \vee \psi) :: (\psi \vee \phi) \\ (\phi \bullet \psi) :: (\psi \bullet \phi) \end{array}$$

## 2.4 Duplication (Dup)

$$\begin{array}{l} (\phi \vee \phi) :: \phi \\ (\phi \bullet \phi) :: \phi \end{array}$$

## 2.5 Association (Assoc)

$$\begin{array}{l} (\phi \vee \psi) \vee \xi :: \phi \vee (\psi \vee \xi) \\ (\phi \bullet \psi) \bullet \xi :: \phi \bullet (\psi \bullet \xi) \end{array}$$

## 2.6 Biconditional exchange (BE)

$$(\phi \equiv \psi) :: (\phi \supset \psi) \bullet (\psi \supset \phi)$$

## 2.7 De Morgan's (DeM)

$$\begin{array}{l} \sim (\phi \vee \psi) :: \sim \phi \bullet \sim \psi \\ \sim (\phi \bullet \psi) :: \sim \phi \vee \sim \psi \end{array}$$

## 2.8 Exportation (Exp)

$$\{(\phi \bullet \psi) \supset \xi\} :: \{\phi \supset (\psi \supset \xi)\}$$

## 2.9 Distribution (Dist)

$$\begin{array}{l} \{\phi \vee (\psi \bullet \xi)\} :: \{(\phi \vee \psi) \bullet (\phi \vee \xi)\} \\ \{\phi \bullet (\psi \vee \xi)\} :: \{(\phi \bullet \psi) \vee (\phi \bullet \xi)\} \end{array}$$

## 2.10 Contraposition (Cont)

$$(\phi \supset \psi) :: (\sim \psi \supset \sim \phi)$$