Truth functions, evaluating compound statements

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1. Functions, arithmetic functions, and truth functions

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2. Definitions of truth functions

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3. Evaluating compound expressions

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4. Categorizing statements (contingencies, tautologies, contradictions)

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3. Evaluating compound expressions

4. Categorizing statements (contingencies, tautologies, contradictions)

5. Relations between statements (equivalence, consistency, implication, validity)

Functions

Functions

A function is something that takes inputs and produces outputs.

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You can think of it as a sort of abstract machine - like a bread machine that will produce as 'output' bread, if it is given as 'input' flour, yeast, sugar, etc.

Arithmetic Functions

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Familiar examples of functions are the arithmetic functions addition, subtraction, multiplication and division

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Addition takes 2 numbers as input, and produces 1 number as output

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If you input 4 and 3, it outputs 7

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A function can be defined in terms of its entire input-output structure

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Input 1 Input 2 Output

x	y	x+y
1	1	2
1	2	3
2	1	3
2	2	4

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Addition ‘+’

x	y	x+y
1	1	2
1	2	3
2	1	3
2	2	4

A function can be defined in terms of its entire input-output structure

Addition ‘+’

x	y	x+y
1	1	2
1	2	3
2	1	3
2	2	4

Rickification ‘∧’

x	y	x∧y
1	1	2
1	2	3
2	1	3
2	2	4

Addition ‘+’

x	y	x+y
1	1	2
1	2	3
2	1	3
2	2	4

Subtraction ‘-’

x	y	x-y
1	1	0
1	2	-1
2	1	1
2	2	0

Multiplication ‘x’

x	y	x x y
1	1	1
1	2	2
2	1	2
2	2	4

Division ‘+’

x	y	x÷y
1	1	1
1	2	0.5
2	1	2
2	2	1

Truth Functions

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- True (T)
- False (F)

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- True (T)
- False (F)

Every atomic statement is either True or False

The statement operators that form compound statements (conjunction, negation, etc.) symbolize truth functions.

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C = I left you my car.

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I left you my house and I left you my car. (H • C)

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H	C	H • C
T	T	

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I left you my house and I left you my car. (H • C)

H	C	H • C
T	T	T
T	F	F
F	T	F
F	F	F

Five truth functions

Conjunction

Conjunction

φ	ψ	φ•ψ
T	T	T
T	F	F
F	T	F
F	F	F

Negation

Negation

I am married.

I am not married.

Negation

I am married.

I am not married.

φ	~φ
T	

Negation

I am married.

I am not married.

φ	~φ
T	F

Negation

I am married.

I am not married.

φ	~φ
T	F
F	

Negation

I am married.

I am not married.

φ	~φ
T	F
F	T

Disjunction

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Shelly won Lotto

Shelly got a big inheritance

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Either Shelly won Lotto, or Shelly got a big inheritance.

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φ	ψ	φ v ψ
T	T	

Disjunction

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T	T	T

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T	T	T
T	F	

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T	T	T
T	F	T
F	T	

Disjunction

Shelly won Lotto

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Shelly either won Lotto or got a big inheritance.

φ	ψ	φ v ψ
T	T	T
T	F	T
F	T	T

Disjunction

Shelly won Lotto

Shelly got a big inheritance

Shelly either won Lotto or got a big inheritance.

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	

Disjunction

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Shelly either won Lotto or got a big inheritance.

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

The expression ‘or’ in English is ambiguous, can express two different truth functions.

You can have soup or salad.

φ	ψ	$\varphi \veebar \psi$
T	T	F
T	F	T
F	T	T
F	F	F

Disjunction

The expression ‘or’ in English is ambiguous, can express two different truth functions.

You can have soup or salad.

φ	ψ	$\varphi \veebar \psi$
T	T	F
T	F	T
F	T	T
F	F	F

Disjunction

Inclusive OR

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Inclusive OR

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

φ	ψ	$\varphi \veebar \psi$
T	T	F
T	F	T
F	T	T
F	F	F

Disjunction

Inclusive OR

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

φ	ψ	$\varphi \veebar \psi$
T	T	F
T	F	T
F	T	T
F	F	F

You can have soup or salad.

Disjunction

Inclusive OR

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

φ	ψ	$\varphi \veebar \psi$
T	T	F
T	F	T
F	T	T
F	F	F

You can have soup or salad.

(P v D) ● ~(P ● D)

Conditional

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You turn in all the homework.

I give you an A in the class.

Conditional

You turn in all the homework.

I give you an A in the class.

If you turn in all the homework,
I'll give you an A in the class.

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φ	ψ	$\varphi \supset \psi$
T	T	

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φ	ψ	$\varphi \supset \psi$
T	T	T

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φ	ψ	$\varphi \supset \psi$
T	T	T
T	F	

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T	T	T
T	F	F

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T	T	T
T	F	F
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T	T	T
T	F	F
F	T	T
F	F	

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If you turn in all the homework,
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φ	ψ	$\varphi \supset \psi$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

Biconditional

You earn a C- or better.
I'll give you a P

Biconditional

You earn a C- or better.
I'll give you a P

I'll give you a P if and only if
you earn a C- or better.

Biconditional

You earn a C- or better.
I'll give you a P

I'll give you a P if and only if
you earn a C- or better.

φ	ψ	$\varphi \equiv \psi$
T	T	

Biconditional

You earn a C- or better.
I'll give you a P

I'll give you a P if and only if
you earn a C- or better.

φ	ψ	$\varphi \equiv \psi$
T	T	T

Biconditional

You earn a C- or better.
I'll give you a P
I'll give you a P if and only if you earn a C- or better.

φ	ψ	$\varphi \equiv \psi$
T	T	T
T	F	

Biconditional

You earn a C- or better.
I'll give you a P
I'll give you a P if and only if you earn a C- or better.

φ	ψ	$\varphi \equiv \psi$
T	T	T
T	F	F

Biconditional

You earn a C- or better.
I'll give you a P
I'll give you a P if and only if you earn a C- or better.

φ	ψ	$\varphi \equiv \psi$
T	T	T
T	F	F
F	T	

Biconditional

You earn a C- or better.
I'll give you a P
I'll give you a P if and only if you earn a C- or better.

φ	ψ	$\varphi \equiv \psi$
T	T	T
T	F	F
F	T	F

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I'll give you a P
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φ	ψ	$\varphi \equiv \psi$
T	T	T
T	F	F
F	T	F
F	F	

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I'll give you a P if and only if you earn a C- or better.

φ	ψ	$\varphi \equiv \psi$
T	T	T
T	F	F
F	T	F
F	F	T

Evaluating compound statements

(P • Q) v (Q ≡ P)

(P • Q) v (Q ≡ P)

Construct a Truth Table

Number of Rows = 2^n

Where n is the number of atomic statements involved: 2^2 = 4 [plus one on top]

(P • Q) v (Q ≡ P)

Construct a Truth Table

Number of Columns: 1 for each statement involved in the compound statement.

P; Q; P • Q; Q ≡ P; (P • Q) v (Q ≡ P) = 5

(P • Q) v (Q ≡ P)

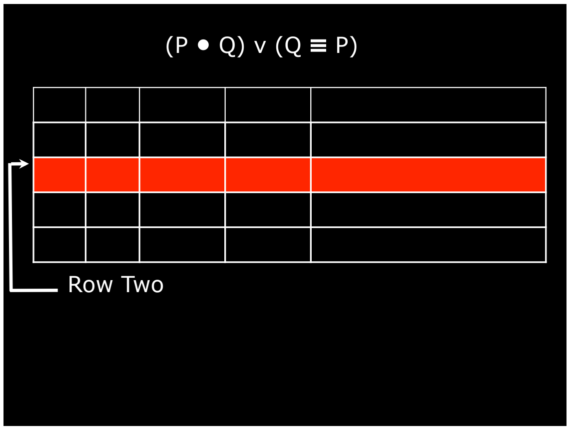
Construct a Truth Table

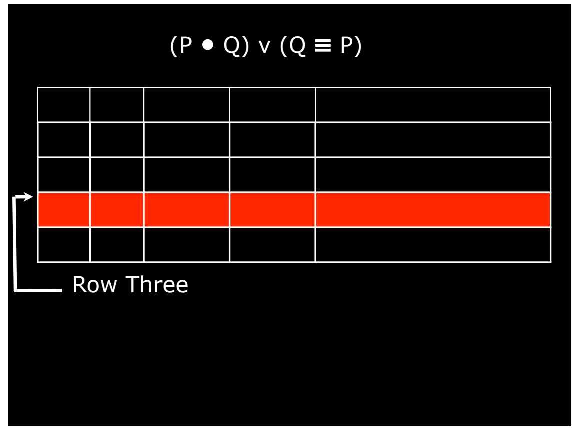
Number of Columns: 1 for each statement involved in the compound statement.

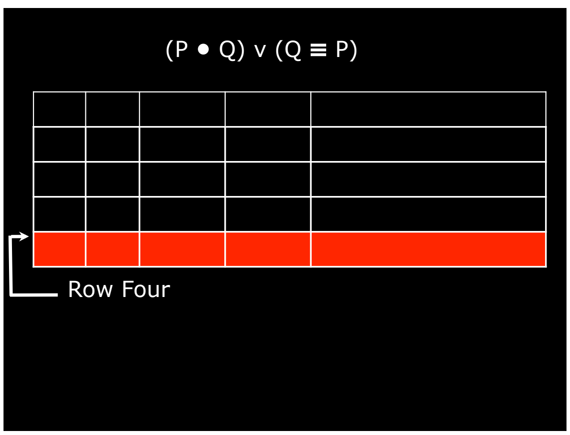
P; Q; P • Q; Q ≡ P; (P • Q) v (Q ≡ P) = 5

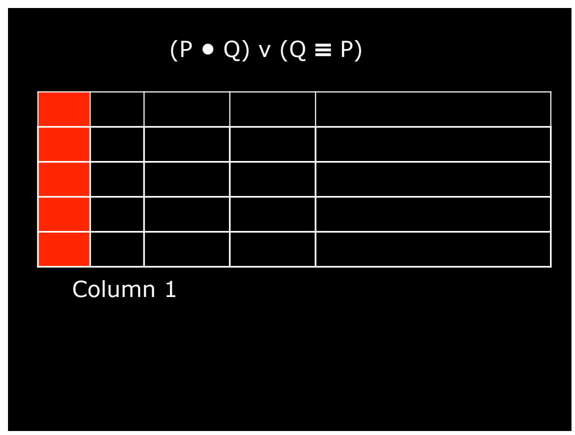
(P • Q) v (Q ≡ P)

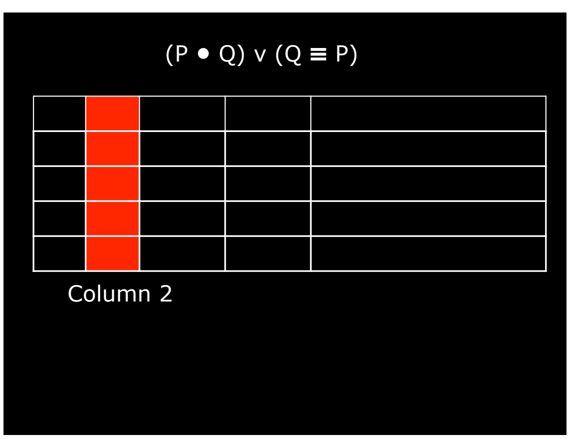
Row One

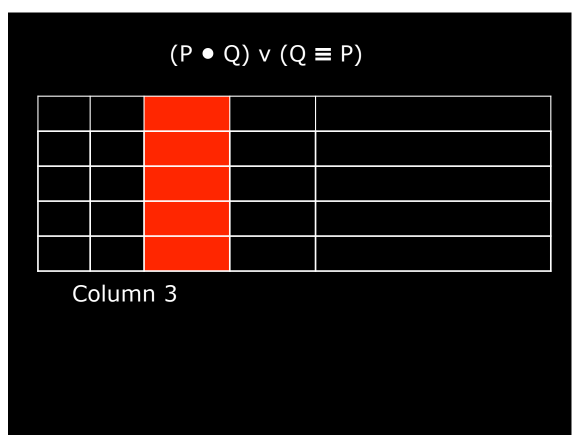

(P • Q) v (Q ≡ P)
Row Two


(P • Q) v (Q ≡ P)
Row Three


(P • Q) v (Q ≡ P)
Row Four


(P • Q) v (Q ≡ P)
Column 1


(P • Q) v (Q ≡ P)
Column 2


(P • Q) v (Q ≡ P)
Column 3

$(P \bullet Q)\ v\ (Q \equiv P)$

Column 4

$(P \bullet Q)\ v\ (Q \equiv P)$

Column 5

$(P \bullet Q)\ v\ (Q \equiv P)$

P	Q			

In the first columns on the "top" row (which is above Row 1), put the atomic statement in alphabetical order

$(P \bullet Q)\ v\ (Q \equiv P)$

P	Q			
T	T			
T	F			
F	T			
F	F			

In the columns under the atomic statements, fill out Ts and Fs so that every possible combination of truth values has a row

$(P \bullet Q)\ v\ (Q \equiv P)$

P	Q			
T	T			
T	F			
F	T			
F	F			

In the first column, top half Ts and bottom half Fs.
For each subsequent column, alternate groups of Ts and Fs half the size of the groups of the previous column, until finished.

$(P \bullet Q)\ v\ (Q \equiv P)$

P	Q			
T	T			
T	F			
F	T			
F	F			

In the first column, top half Ts and bottom half Fs.
For each subsequent column, alternate groups of Ts and Fs half the size of the groups of the previous column, until finished.

(P • Q) v (Q ≡ P)

P	Q	P • Q	Q ≡ P	(P • Q) v (Q ≡ P)
T	T			
T	F			
F	T			
F	F			

In subsequent columns, build up from the smallest compound statements to the largest, and fill in its truth column according to the truth function of that operator, and the truth values of the components

(P • Q) v (Q ≡ P)

P	Q	P • Q	Q ≡ P	(P • Q) v (Q ≡ P)
T	T	T		
T	F	F		
F	T	F		
F	F	F		

φ	ψ	φ•ψ
T	T	T
T	F	F
F	T	F
F	F	F

(P • Q) v (Q ≡ P)

P	Q	P • Q	Q ≡ P	(P • Q) v (Q ≡ P)
T	T	T	T	
T	F	F	F	
F	T	F	F	
F	F	F	T	

φ	ψ	φ≡ψ
T	T	T
T	F	F
F	T	F
F	F	T

(P • Q) v (Q ≡ P)

P	Q	P • Q	Q ≡ P	(P • Q) v (Q ≡ P)
T	T	T	T	T
T	F	F	F	F
F	T	F	F	F
F	F	F	T	T

φ	ψ	φvψ
T	T	T
T	F	T
F	T	T
F	F	F

~(~C ≡ D) ⊃ ~**D**

~(~C ≡ D) ⊃ ~**D**

$\sim(\sim C \equiv D) \supset \sim\mathbf{D}$

C	D					
T	T					
T	F					
F	T					
F	F					

$\sim(\sim C \equiv D) \supset \sim\mathbf{D}$

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T					
T	F					
F	T					
F	F					

$\sim(\sim C \equiv D) \supset \sim\mathbf{D}$

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F				
T	F	F				
F	T	T				
F	F	T				

$\sim(\sim C \equiv D) \supset \sim\mathbf{D}$

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

$\sim(\sim C \equiv D) \supset \sim\mathbf{D}$

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F		
T	F	F	T	T		
F	T	T	F	T		
F	F	T	T	F		

$\sim(\sim C \equiv D) \supset \sim\mathbf{D}$

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	
T	F	F	T	T	F	
F	T	T	F	T	F	
F	F	T	T	F	T	

$\sim(\sim C \equiv D) \supset \sim D$

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

$\sim R \supset R$

$\sim R \supset R$

$\sim R \supset R$

R		

$\sim R \supset R$

R		
T		
F		

$\sim R \supset R$

R	~R	~R ⊃ R
T		
F		

~R ⊃ R

R	~R	~R ⊃ R
T	F	
F	T	

~R ⊃ R

R	~R	~R ⊃ R
T	F	T
F	T	F

φ	ψ	φ⊃ψ
T	T	T
T	F	F
F	T	T
F	F	T

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T									
T									
T									
T									
F									
F									
F									
F									

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T								
T	T								
T	F								
T	F								
F	T								
F	T								
F	F								
F	F								

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T	F						
T	T	F	F						
T	F	T	F						
T	F	F	F						
F	T	T	T						
F	T	F	T						
F	F	T	T						
F	F	F	T						

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T	F	F					
T	T	F	F	T					
T	F	T	F	F					
T	F	F	F	T					
F	T	T	T	F					
F	T	F	T	T					
F	F	T	T	F					
F	F	F	T	T					

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T	F	F	T				
T	T	F	F	T	T				
T	F	T	F	F	T				
T	F	F	F	T	T				
F	T	T	T	F	T				
F	T	F	T	T	T				
F	F	T	T	F	F				
F	F	F	T	T	F				

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T	F	F	T	F			
T	T	F	F	T	T	T			
T	F	T	F	F	T	F			
T	F	F	F	T	T	T			
F	T	T	T	F	T	F			
F	T	F	T	T	T	T			
F	F	T	T	F	F	T			
F	F	F	T	T	F	T			

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T	F	F	T	F	F		
T	T	F	F	T	T	T	F		
T	F	T	F	F	T	F	F		
T	F	F	F	T	T	T	F		
F	T	T	T	F	T	F	F		
F	T	F	T	T	T	T	T		
F	F	T	T	F	F	T	F		
F	F	F	T	T	F	T	T		

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T	F	F	T	F	F	T	
T	T	F	F	T	T	T	F	T	
T	F	T	F	F	T	F	F	T	
T	F	F	F	T	T	T	F	T	
F	T	T	T	F	T	F	F	T	
F	T	F	T	T	T	T	T	F	
F	F	T	T	F	F	T	F	T	
F	F	F	T	T	F	T	T	F	

{(G v E) ⊃ ~K} ⊃ ~(~K • ~E)

E	G	K	~E	~K	GvE	(GvE) ⊃~K	~K• ~E	~(~K• ~E)	{(GvE)⊃~K} ⊃~(~K•~E)
T	T	T	F	F	T	F	F	T	T
T	T	F	F	T	T	T	F	T	T
T	F	T	F	F	T	F	F	T	T
T	F	F	F	T	T	T	F	T	T
F	T	T	T	F	T	F	F	T	T
F	T	F	T	T	T	T	T	F	F
F	F	T	T	F	F	T	F	T	T
F	F	F	T	T	F	T	T	F	F

What are truth tables doing exactly?

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We have a compound statement, composed of atomic statements.

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So we list off all the possibilities

Truth table shows, for each possibility, whether the compound statement(s) is/are true or false.

~(~C ≡ D) ⊃ ~D

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

~(~C ≡ D) ⊃ ~D

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders   D = Dogs are robots

~(~C ≡ D) ⊃ ~D

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders   D = Dogs are robots

If it's not true that cats don't eat spiders if and only if dogs are robots, then dogs aren't robots.

~(~C ≡ D) ⊃ ~D

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders   D = Dogs are robots

Row one represents the alternate universe in which Cats eat spiders and dogs are robots. In that universe, the compound statement is False.

~(~C ≡ D) ⊃ ~D

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders   D = Dogs are robots

Row two represents the alternate universe in which Cats eat spiders and dogs aren't robots. In that universe, the compound statement is True.

~(~C ≡ D) ⊃ ~**D**

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders D = Dogs are robots

Row three represents the alternate universe in which Cats don't eat spiders and dogs are robots. In that universe, the compound statement is True.

~(~C ≡ D) ⊃ ~**D**

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders D = Dogs are robots

Row four represents the alternate universe in which Cats don't eat spiders and dogs aren't robots. In that universe, the compound statement is True.

~(~C ≡ D) ⊃ ~**D**

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders D = Dogs are robots

Now we can know things like: any universe in which dogs aren't robots is a universe in which the compound statement is True.

~(~C ≡ D) ⊃ ~**D**

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

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~(~C ≡ D) ⊃ ~**D**

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders D = Dogs are robots

It turns out that the real universe is one of the possible alternate universes. In fact, on this truth table, it is represented on Row 2, since in fact cats do eat spiders, and dogs aren't robots.

~(~C ≡ D) ⊃ ~**D**

C	D	~C	~D	~C≡D	~(~C≡D)	~(~C≡D)⊃ ~D
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

C = Cats eat spiders D = Dogs are robots

It turns out that the real universe is one of the possible alternate universes. In fact, on this truth table, it is represented on Row 2, since in fact cats do eat spiders, and dogs aren't robots.

Categorizing statements

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It is often useful to know the range of truth values that a statement can have.

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Statements that are true in all alternate universes are tautologies.

Statements that are false in all alternate universes are contradictions.

Statements that are true in some universes and false in others are contingencies.

Categorizing statements

By definition, atomic statements are contingencies: they can be either true or false. This does not mean that all compound statements are contingencies, though.

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P v ~P

Categorizing statements

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P v ~P

P	~P	P v ~P
T	F	T
F	T	T

To determine whether a statement is a tautology, contradiction, or contingency, do a truth table for that statement, and look at its truth column:

- if it has only Ts, then it is a tautology
- if it has only Fs, then it is a contradiction
- if it has both Ts and Fs, then it is a contingency

P v ~P

P	~P	P v ~P
T	F	T
F	T	T

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- if it has only Ts, then it is a tautology
- if it has only Fs, then it is a contradiction
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P v ~P

P	~P	P v ~P
T	F	T
F	T	T

B • ~B

B • ~B

B	~B	B • ~B

B • ~B

B	~B	B • ~B
T		
F		

B • ~B

B	~B	B • ~B
T	F	
F	T	

B • ~B

B	~B	B • ~B
T	F	F
F	T	F

R • ~(S v R)

R • ~(S v R)

R • ~(S v R)

R	S	S v R	~(S v R)	R • ~(S v R)

R • ~(S v R)

R	S	S v R	~(S v R)	R • ~(S v R)
T	T			
T	F			
F	T			
F	F			

R • ~(S v R)

R	S	S v R	~(S v R)	R • ~(S v R)
T	T	T		
T	F	T		
F	T	T		
F	F	F		

R • ~(S v R)

R	S	S v R	~(S v R)	R • ~(S v R)
T	T	T	F	
T	F	T	F	
F	T	T	F	
F	F	F	T	

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R	S	S v R	~(S v R)	R • ~(S v R)
T	T	T	F	F
T	F	T	F	F
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Equivalence, consistency, implication, & validity

Equivalence, consistency, implication, & validity

1. Relations between two statements: equivalence, consistency, implication

Equivalence, consistency, implication, & validity

1. Relations between two statements: equivalence, consistency, implication

2. Relations between three or more statements: equivalence, consistency, joint implication

Equivalence, consistency, implication, & validity

1. Relations between two statements: equivalence, consistency, implication

2. Relations between three or more statements: equivalence, consistency, joint implication

3. Argument validity

Relations between two statements

Relations between two statements

Just as it is often helpful to know whether an individual statement is a tautology, contradiction, or contingency, it is also often helpful to know what relations hold between two statements.

Equivalence

Two statements are equivalent iff they have identical truth columns.

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No matter what universe you are in, the statements will have the same truth value.

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Two statements are equivalent iff they have identical truth columns.

No matter what universe you are in, the statements will have the same truth value.

To test for equivalence, construct a joint truth table for the two statements and compare their truth columns. If the columns are identical, then the statements are equivalent. If they are not identical, then they are not equivalent.

Consistency

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There is some universe in which the statements are both True.

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Two statements are consistent iff it is possible for them to both be true at the same time.

There is some universe in which the statements are both True.

To test for consistency, do a joint truth table for the two statements. If there is a row (one or more) on which both statements are T, then they are consistent. If there is no row in which both are T, then they are inconsistent.

Implication

Statement φ implies statement ψ iff:

if φ is true, then ψ must be true.

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Statement φ implies statement ψ iff:

if φ is true, then ψ must be true.

In any universe where φ is true, ψ will also be true.

In any universe where ψ is false, φ will also be false.

Implication

Statement φ implies statement ψ iff:

if φ is true, then ψ must be true.

To see whether φ implies ψ, do a joint truth table for φ and ψ, and look for a row on which φ is T and ψ is F (a counter-example row). If there IS a counter-example row, then then φ does NOT imply ψ; if there is NOT a counter-example row, then φ DOES imply ψ.

1. ~(P • Q) **2.** ~P • ~Q

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Are (1) and (2) equivalent?

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Are (1) and (2) equivalent? No

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
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Are (1) and (2) consistent?

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Are (1) and (2) consistent? Yes

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Does (1) imply (2)?

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Does (1) imply (2)? No

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Does (2) imply (1)?

1. ~(P • Q) **2.** ~P • ~Q

P	Q	~P	~Q	P • Q	~(P • Q)	~P • ~Q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Does (2) imply (1)? Yes

1. A ≡ B **2.** (A ⊃ B) • (B ⊃ A)

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A	B	A⊃B	B⊃A	A≡B	(A ⊃ B)•(B ⊃ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

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T	F	F	T	F	F
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T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

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A	B	A⊃B	B⊃A	A≡B	(A ⊃ B)•(B ⊃ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Does (2) imply (1)?

1. A ≡ B **2.** (A ⊃ B) • (B ⊃ A)

A	B	A⊃B	B⊃A	A≡B	(A ⊃ B)•(B ⊃ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Does (2) imply (1)? Yes

1. (M ⊃ N) • ~N **2.** ~M

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M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

1. (M ⊃ N) • ~N **2.** ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Are (1) and (2) equivalent?

1. (M ⊃ N) • ~N **2.** ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

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M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

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M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

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M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Does (1) imply (2)?

1. (M ⊃ N) • ~N **2.** ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Does (1) imply (2)? Yes

1. (M ⊃ N) • ~N **2.** ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Does (2) imply (1)?

1. (M ⊃ N) • ~N **2.** ~M

M	N	M ⊃ N	~N	(M ⊃ N) • ~N	~M
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	T	T

Does (2) imply (1)? No

Relations between three or more statements

Equivalence

A set of statements are equivalent if they all have identical truth columns

Consistency

A set of statements are consistent if there is at least one row on which they are all T.

Joint Implication

A set of statements ($\varphi_1 \ldots \varphi_n$) jointly imply statement ψ iff: if ($\varphi_1 \ldots \varphi_n$) are all true, then ψ must be true.

Joint Implication

A set of statements (φ_1 ... φ_n) jointly imply statement ψ iff: if (φ_1 ... φ_n) are all true, then ψ must be true.

To test whether statements (φ_1 ... φ_n) jointly imply statement ψ, do a joint truth table with all statements, and look for a row in which ALL of (φ_1 ... φ_n) are T and ψ is F (a CE row). If there IS such a row, then (φ_1 ... φ_n) do NOT jointly imply ψ; if there is NOT such a row, then (φ_1 ... φ_n) DO jointly imply ψ.

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

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2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G **v** H
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

				1	2	3
G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G **v** H
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

				1	2	3
G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G **v** H
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Are 1-3 equivalent?

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

				1	2	3
G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G **v** H
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Are 1-3 equivalent? Yes

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G v H
				1	2	3
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Are 1-3 consistent?

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G v H
				1	2	3
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Are 1-3 consistent? Yes

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G v H
				1	2	3
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Do 1-2 jointly imply 3?

1. G ⊃ H
2. ~H ⊃ ~G
3. ~G v H

G	H	~G	~H	G ⊃ H	~H ⊃ ~G	~G v H
				1	2	3
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Do 1-2 jointly imply 3? Yes

1. L
2. L ⊃ K
3. ~K

1. L
2. L ⊃ K
3. ~K

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

1. L
2. L ⊃ K
3. ~K

1 2 3

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

1. L
2. L ⊃ K
3. ~K

1 2 3

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Are 1-3 equivalent?

1. L
2. L ⊃ K
3. ~K

1 2 3

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Are 1-3 equivalent? No

1. L
2. L ⊃ K
3. ~K

1 2 3

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Are 1-3 consistent?

1. L
2. L ⊃ K
3. ~K

1 2 3

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Are 1-3 consistent? No

1. L
2. L ⊃ K
3. ~K

1 2 3

K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 1-2 jointly imply 3?

1. L
2. L ⊃ K
3. ~K

		1	2	3
K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 1-2 jointly imply 3? No

1. L
2. L ⊃ K
3. ~K

		1	2	3
K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 1&3 jointly imply 2?

1. L
2. L ⊃ K
3. ~K

		1	2	3
K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 1&3 jointly imply 2? No

1. L
2. L ⊃ K
3. ~K

		1	2	3
K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 2&3 jointly imply 1?

1. L
2. L ⊃ K
3. ~K

		1	2	3
K	L	L ⊃ K	~K
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	T

Do 2&3 jointly imply 1? No

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

	4		1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

	4		1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Are 1-4 equivalent?

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

	4		1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Are 1-4 equivalent? No

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

	4		1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Are 1-4 consistent?

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

	4		1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Are 1-4 consistent? Yes

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

4			1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Do 1-3 jointly imply 4?

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

4			1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Do 1-3 jointly imply 4? Yes

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

4			1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Do 1, 2 & 4 jointly imply 3?

1. A ⊃ B
2. C ⊃ B
3. A v C
4. B

4			1	2	3
A	B	C	A ⊃ B	C ⊃ B	A v C
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	T	F

Do 1, 2 & 4 jointly imply 3? No

Argument validity

An argument is deductively valid iff the premises jointly imply the conclusion.

1. P v C
2. ~C
∴ **3.** P

1. P v C
2. ~C
∴ **3.** P

This pizza is either pepperoni or cheese. It is not cheese. Therefore, it is pepperoni.

1. P v C
2. ~C
∴ **3.** P

C	P	~C	P **v** C
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

This pizza is either pepperoni or cheese. It is not cheese. Therefore, it is pepperoni.

1. P v C
2. ~C
∴ **3.** P

C	P	~C	P **v** C
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

This pizza is either pepperoni or cheese. It is not cheese. Therefore, it is pepperoni.

Argument is valid

1. G
2. L
∴ **3.** D

1. G
2. L
∴ **3.** D

Trees are green. Lead is heavy. Therefore, Elvis is dead.

1. G
2. L
∴ **3.** D

D	G	L
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Trees are green. Lead is heavy. Therefore, Elvis is dead.

1. G
2. L
∴ **3.** D

D	G	L
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Trees are green. Lead is heavy. Therefore, Elvis is dead.

Argument is invalid

1. D ⊃ R
2. D
∴ **3.** R

1. D ⊃ R
2. D
∴ **3.** R

If ducks sink, then ducks are made of small rocks. Ducks do sink. Therefore, ducks are made of small rocks.

1. D ⊃ R
2. D
∴ **3.** R

D	R	D ⊃ R
T	T	T
T	F	F
F	T	T
F	F	T

If ducks sink, then ducks are made of small rocks. Ducks do sink. Therefore, ducks are made of small rocks.

1. D ⊃ R
2. D
∴ **3.** R

D	R	D ⊃ R
T	T	T
T	F	F
F	T	T
F	F	T

If ducks sink, then ducks are made of small rocks. Ducks do sink. Therefore, ducks are made of small rocks.

Argument is valid

1. ~A ⊃ K
2. ~K ≡ ~A
3. ~K
∴ **4.** J

1. ~A ⊃ K
2. ~K ≡ ~A
3. ~K
∴ **4.** J

A	J	K	~A	~K	~A ⊃ K	~K ≡ ~A
T	T	T	F	F	T	T
T	T	F	F	T	T	F
T	F	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	T	F	T

1. ~A ⊃ K
2. ~K ≡ ~A
3. ~K
∴ **4.** J

Argument is valid

A	J	K	~A	~K	~A ⊃ K	~K ≡ ~A
T	T	T	F	F	T	T
T	T	F	F	T	T	F
T	F	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	T	F	T