Bohr, Einstein, and the EPR experiment

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MA Seminar: Philosophy of Physics

Hilbert spaces

Barrett, *The Quantum Mechanics of Minds and Worlds*, Appendix

Quantum states are represented in special vector spaces, so-called Hilbert spaces, which for our purposes are defined as follows:

Definition (Hilbert space)

A Hilbert space $\mathcal{H}$ is a complex-valued vector space with an inner (or scalar) product and with the following additional properties:

1. *$\mathcal{H}$ can be either finite-dimensional or countably infinite dimensional.*
2. *$\mathcal{H}$ is complete, i.e. every Cauchy sequence in $\mathcal{H}$, every sequence where the distance between successive elements in the sequence becomes arbitrarily small, converges to an element of $\mathcal{H}$.*
3. *$\mathcal{H}$ is separable, i.e. there is a countable sequence of elements in $\mathcal{H}$ that is everywhere dense in $\mathcal{H}$.*

- Completeness of $\mathcal{H}$ meant to provide a space rich enough to take limits.
- Separability places a limit on just how large $\mathcal{H}$ can be, since an infinite-dimensional space is separable iff the dimension of the space is countable.
- If a space is separable, we have a unique decomposition of its elements with respect to a chosen basis ⇒ we can make physical sense of the formalism
- but: requiring separability implies that we cannot represent states where a continuously valued physical quantity has determinate properties as elements in $\mathcal{H}$ (Why?)
- Neumann suggested using discrete quantities to represent continuous quantities to the desired degree of precision.

⇒ some of the richness of Dirac's theory is sacrificed for mathematical rigour

Bohr: the correspondence principle

Bohr, "Discussion with Einstein on epistemological problems in atomic physics", 1949.

Principle (Correspondence (CP))

Classical physics and quantum physics must give the same predictions for 'large systems'. More precisely, quantum and classical physics agree in the so-called classical limit, i.e. when the quantum numbers characterizing the system are large.

- matrix mechanics: CP used to construct theory; wave mechanics: Ehrenfest showed that Newton's laws hold on average (i.e. expectation values of position and momentum obey Newton's laws)
- CP constrains construction of QT by quantization recipes in that it gives special status to those operators corresponding to classical magnitudes and identifies incompatible pairs of them (by using conjugate canonical variables of classical Hamiltonian mechanics and their Poisson bracket structure)

Como 1927: complementarity

International Physics Congress, Como (Italy), September 1927

> *"...it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms...*
>
> *"This crucial point... implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear. In fact, the individuality of the typical quantum effects finds its proper expression in the circumstance that any attempt of subdividing the phenomena will demand a change in the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled.*

“Consequently, evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects.

“Under these circumstances an essential element of ambiguity is involved in ascribing conventional physical attributes to atomic objects, as is at once evident in the dilemma regarding the corpuscular and wave properties of electrons and photons, where we have to do with contrasting pictures, each referring to an essential aspect of empirical evidence.” (op. cit., 209f)

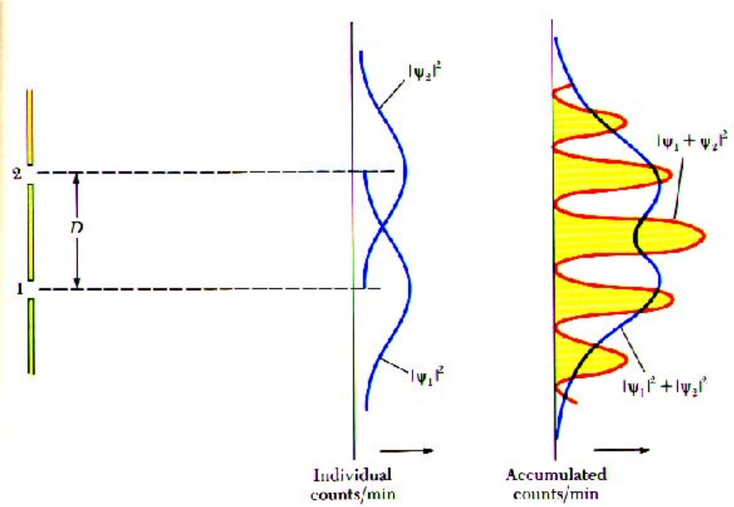
Principle (Complementarity)

A quantum mechanical object can have seemingly contradictory properties, exhibiting different ones in different experiment settings. For example, depending on the experimental set-up, a single quantum-mechanical object can either behave in a particle-like or a wave-like manner, but not simultaneously as both. The particle-like and the wave-like behaviour, for which the particle has a real propensity or disposition, are mutually exclusive in the sense that they can never be observed simultaneously.

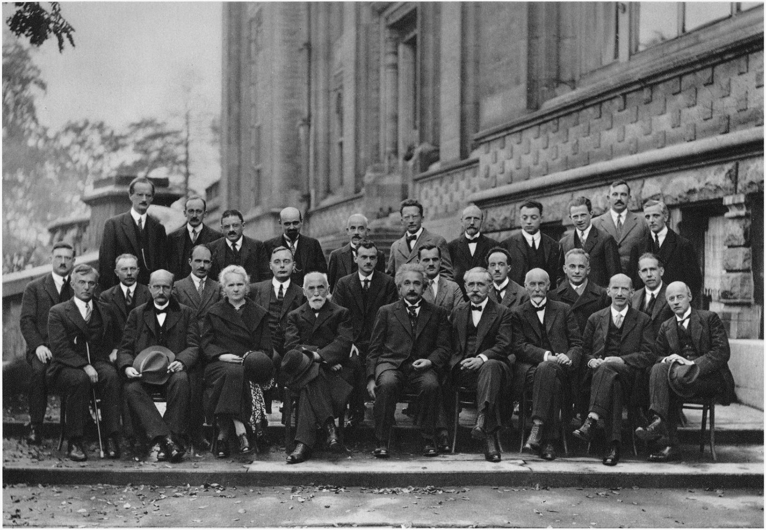
Comments:

- Complementarity does not merely restrict what we can know about the properties of a physical entity, but it imposes limits to that entity's very manifestation of the property in the physical world.
- All properties of a quantum-mechanical system come in complementary pairs, corresponding to pairs of incompatible properties as declared by the uncertainty relation.
- Principle of complementarity $\neq$ uncertainty relation

Illustration: double slit experiment



Solvay Conference, Brussels, October 1927



The famous Einstein-Bohr debates

Walter Isaacson, *Einstein: His Life and Universe*, Simon and Schuster, 2007.

Einstein tried to prove that QM did not give a complete description of reality, using thought experiments involving various contraptions.

> *"For example, one of Einstein's thought experiments involved a beam of electrons that is sent through a slit in a screen, and then the position of the electrons are recorded as they hit a photographic plate. Various other elements, such as a shutter to open and close the slit instantaneously, were posited by Einstein in his ingenious efforts to show that position and momentum could in theory be known with precision.*
>
> *" 'Einstein would bring along to breakfast a proposal of this kind,' Heisenberg recalled...*

“The group would usually make their way to the Congress hall together, working on ways to refute Einstein’s problem. ‘By dinner-time we could usually prove that his thought experiment did not contradict uncertainty relations,’ Heisenberg recalled, and Einstein would concede defeat. ‘But the next morning he would bring along to breakfast a new thought experiment, generally more complicated than the previous one.’ By dinnertime that would be disproved as well.

“Back and forth they went, each lob from Einstein volleyed back by Bohr, who was able to show how the uncertainty principle, in each instance, did indeed limit the amount of knowable information about a moving electron. ‘And so it went for several days,’ said Heisenberg. ‘In the end, we—that is, Bohr, Pauli, and I—knew that we could now be sure of our ground.’ ” (p. 346)

In Bohr's view, all of Einstein's efforts failed in proving that QM was incomplete or otherwise lacking:

> *"In my opinion, there could be no other way to deem a logically consistent mathematical formalism as inadequate than by demonstrating the departure of its consequences from experience or by proving that its predictions did not exhaust the possibilities of observation, and Einstein's argumentation could be directed to neither of these ends." (Op. cit., p. 229)*

Einstein's opposition to QM

Arthur Fine, *The Shaky Game: Einstein Realism and the Quantum Theory*, U of Chicago Press, 1986, 1996; p. 31.

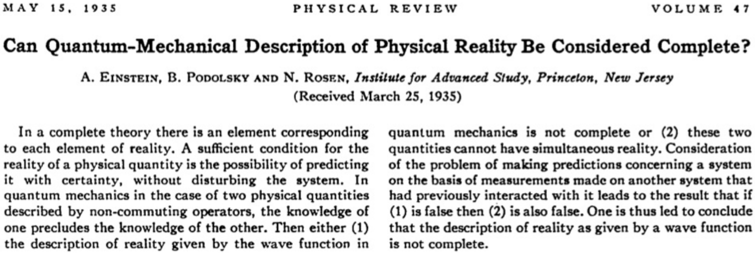
According to Arthur Fine, Einstein took issue with QM on four counts:

- Concerning external constraints imposed by other theories:
    1. how to reconcile QM with the requirements of relativity
    2. how to get satisfactory classical approximations from QM
- Concerning central problems in the interpretation of the theory itself:
    3. distant correlations and action-at-a-distance
    4. issue of statistics and the description of individual systems

Einstein-Podolsky-Rosen 1935

Boris Podolsky (1896-1966), Nathan Rosen (1909-1995)

A. Einstein, B. Podolsky, and N. Rosen," Can quantum-mechanical description of physical reality be considered complete?" *Phys. Rev.* **47** (1935): 777-781.



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Completeness and reality

According to EPR, for a theory to be complete, it must satisfy the following necessary condition:

Condition (Completeness)

"Every element of the physical reality must have a counterpart in the physical theory." (777)

A sufficient condition for a physical theory to satisfy a criterion of reality is the following:

Condition (Reality)

"If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." (ibid.)

The argument

- a quantum-mechanical state is supposed to be completely characterized by the wave function $|\psi\rangle$
- eigenstate-eigenvalue link: if $|\psi\rangle$ is an eigenfunction of an operator $\hat{A}$, i.e. if
$$\hat{A}|\psi\rangle = a|\psi\rangle \tag{1}$$
holds, then the physical quantity (associated with) $\hat{A}$ has with certainty the value *a* whenever the systems is in state $|\psi\rangle$
- Reality $\Rightarrow$ if (1) holds for a particle in state $|\psi\rangle$, then there is an element of reality corresponding to physical quantity $\hat{A}$

- From the fact that non-commuting operators don't have the same eigenfunctions, it "follows that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality." (778)
- (because Completeness $\Rightarrow$ if both had reality (and thus definite values) these values would enter the complete description, but that can't simultaneously be the case)
- Although it is initially plausible to think that $|\psi\rangle$ completely describes the state (since it contains all information about all quantities that can be measured without changing the state), this assumption, together with Reality, leads to a contradiction.

"We see... that, as a consequence of two different measurements performed upon the first [*of two entangled, but spatially separate*] *system*[*s*], *the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems. Thus, it is possible to assign two different wave functions... to the same reality (the second system after the interaction with the first)." (779)*

- Important: the two measurements are of physical quantities whose associated operators do not commute

"Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with noncommuting operators, can have simultaneous reality. Thus the negation of (1) [*QM-description of reality given by* $|\psi\rangle$ *is not complete*] *leads to the negation of the only other alternative (2)* [*noncommuting operators cannot have simultaneous reality*]. *We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete." (780)*

P.S.

"Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously measured or predicted... No reasonable definition of reality could be expected to permit this." (780)

Bohr's reaction to EPR (1935)

N. Bohr, *Phys. Rev.* **48** (1935): 696.

> *"Such an argumentation, however, would hardly seem suited to affect the soundness of quantum-mechanical description, which is based on a coherent mathematical formalism covering automatically any procedure of measurement like that indicated. The apparent contradiction in fact discloses only an essential inadequacy of the customary viewpoint of natural philosophy for a rational account of physical phenomena of the type with which we are concerned in quantum mechanics.*

"Indeed the finite interaction between object and measuring agencies conditioned by the very existence of the quantum of action entails—because of the impossibility of controlling the reaction of the object on the measuring instruments, if these are to serve their purpose—the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality. In fact, as we shall see, a criterion of reality like that proposed by the named authors contains—however cautious its formulation may appear—an essential ambiguity when it is applied to the actual problems with which we are here concerned." (696f)

"From our point of new we now see that the wording of the above-mentioned criterion of physical reality proposed by Einstein, Podolsky, and Rosen contains an ambiguity as regards the meaning of the expression 'without in any way disturbing a system.' Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. [*Fine: what?!? Significant departure from Bohr's earlier views*] *But even at this stage there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system.* [*Fine: positivism alert!*] *Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete.*

"On the contrary, this description, as appears from the preceding discussion, may be characterised as a rational utilisation of all possibilities of unambiguous interpretation of measurements, compatible with the finite and uncontrollable interaction between the objects and the measuring instruments in the field of quantum theory. In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena that the notion of complementarity aims at characterising." (700)

The EPR-paradox revisited

- A source creates spin-1/2-particles (such as e^-) in a singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \tag{2}$$

  which are then separated s.t. one e^- moves to left wing, and the other to the right wing.
- Important: spins cancel, total spin is zero

⇒ If L particle is found in 'up' state, then R particle must be in 'down' state (and vice versa).

- In classical physics, that would not be a problem, since we would just conclude that R particle always had spin 'down' from the time of separation.

- However, according to (the standard interpretation of) QM, the spin of the L particle has no definite value until measured.

⇒ When it is measured, it must produce an instantaneous effect in the R wing, collapsing the wave function s.t. the R particle has definite spin too.

⇒ either spooky action-at-a-distance or faster-than-light signalling (⇒ violation of special relativity)

- EPR: this shows that there must be hidden elements of reality ('hidden variables'), which QM fails to take into account, i.e. QM state description is incomplete

Locality

Principle (Einstein locality)

If two systems are in isolation from each other s.t. they don't interact anymore, then a measurement on the first does not have any real effect on the second.

- Bohr: Einstein locality is violated, the QM-system consists of both particles (and the observer), until a measurement is made

⇒ EPR-paradox doesn't show that QM is incomplete, but only that Einstein locality is violated

Fine's reconstruction of the EPR argument

Fine, pp. 32ff

(INC) The QM description of a system given by the state function is incomplete.

(NSV) Observables represented by noncommuting operators cannot have simultaneous reality.

The argument then proceeds in two demonstrations:

1. (INC) $\vee$ (NSV)
2. $\neg$ (INC) $\rightarrow$ $\neg$ (NSV)

from which it is concluded that (INC) must hold.

The first part

- In order to establish the first claim, EPR show that $\neg$ (NSV) $\rightarrow$ (INC)
- But this part is easy, since, noting that no state can be an eigenstate for noncommuting operators, if a pair of noncommuting operators had simultaneous values, then the state function would incomplete.

The second part

- Assume ¬ (INC) (i.e. that theory is complete) and try to establish the existence of simultaneous values for position and momentum of two particles flying off in opposite directions after having interacted (s.t. momentum was conserved).
- We can predict, from measurement of position of one system, the position of the other; and similarly with respect to momentum.
- If the two particles are sufficiently far apart, measurements on one system will not disturb the other.
- REALITY ⇒ at least one particle must have definite position AND momentum.
- Note that assumption of completeness is never actually used: EPR simply derive ¬ (NSV), which would have allowed for a simpler argument structure!

Fine's comments

- actual argument less clear
- "Finally it is by no means clear how, even with the stated criterion of reality, the fact that one can assign either a definite position or a definite momentum to the unmeasured particle establishes that the particle has both properties at once." (34)
- In later years, Einstein stated the paradox more clearly as an incompatibility between the separability of subsystems and completeness:

  "the paradox forces us to relinquish one of the following two assertions:

  (1) the description by means of the ψ-function is complete

  (2) the real states of spatially separated objects are independent of each other." (in Schilpp (1949), 682)