Bell's theorem and nonlocality

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MA Seminar: Philosophy of Physics

Bertlmann's socks and Bell's theorem

J.S. Bell, 'Bertlmann's socks and the nature of reality', in *Speakable and Unspeakable in QM*, 139-158.

"Dr. Bertlmann likes to wear two socks of different colours. Which colour he will have on a given foot on a given day is quite unpredictable. But when you see... that the first sock is pink you can already be sure that the second sock will not be pink. Observation of the first, and experience of Bertlmann, gives immediate information about the second. There is no accounting for tastes, but apart from that there is no mystery here. And is not the EPR business just the same?" (139)

No, since many physicists

> *"... came to hold not only that it is difficult to find a* [*classical explanation of the EPR business*] *but that it is wrong to look for one—if not actually immoral then certainly unprofessional. Going further still, some asserted that atomic and subatomic particles do not have any definite properties in advance of observation... It is as if we had come to deny the reality of Bertlmann's socks, or at least of their colours, when not looked at. And as if a child has asked: How come they always choose different colours when they are looked at? How does the second sock know what the first had done?" (142f)*

- Bell goes on to use the example of pairs of socks, of which we want to know what the probabilities are that they survive a thousand washing cycles at a certain temperature. (Sec. 3)
- Using a random sampling hypothesis (148), and the fact that socks are paired à la Bertlmann (ibid.), he derives an inequality (a Bell inequality) which can be shown to be violated in QM. (149)

"The EPRB correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting...

"But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we cannot dismiss intervention on one side as a causal influence on the other." (149f)

- Bell then proceeds to generalize the argument in several respects, to show that "certain particular correlations, realizable according to quantum mechanics, are locally inexplicable." (151f) This means that they "cannot be explained... without action at a distance." (152)

Bell sees at least four different positions that might be taken with respect to the EPRB business:

1. QM is wrong in sufficiently critical situations. But that's unconvincing in the light of empirical evidence.
2. The detector settings are not independent variables. But this would imply strange conspiracies between spatially distant apparatuses, or our free will is conspiratorially entangled with them or both.
3. Causal influences can go faster than light, perhaps by reintroducing an aether. But this would create formidable challenges...
4. Perhaps there is no reality beyond some 'classical' 'macroscopic' level.

A closer examination of the assumptions of Bell's theorem

G. Grasshoff, S. Portmann, A. Wüthrich, *British Journal for the Philosophy of Science* **56** (2005): 663-680.

- There are many inequivalent sets of assumptions that are sufficient to derive a Bell-type inequality that is violated by QM and experiment.
- Dialectical situation: try to derive Bell inequality from a set of assumptions that is as weak as possible; since we know that Bell inequality is violated, we know that at least one premise must be false
- But which one?!?
- Traditionally, apart from a number of auxiliary assumptions, or assumptions that come directly from QM, what is often called Bell locality is assumed.
- Nomenclature: L_i, R_j are the settings of the apparatus in the left and right wings, respectively ($i, j = 1, 2, 3$); L_i^a, R_j^b are the possible outcomes measured in the left and right wings, given settings as indicated by i, j, where $a, b = \uparrow, \downarrow$; V is a common cause variable

Assumption (Bell locality)

$$p(L_i^a \wedge R_j^b | V \wedge L_i \wedge R_j) = p(L_i^a | V \wedge L_i)\, p(R_j^b | V \wedge R_j).$$

This condition can be unpacked into several weaker ones, such as

Assumption (Separability)

The coinciding instances of L_i^a and R_j^b are distinct events.

Assumption (Locality 1)

No L_i^a or R_j^b is causally relevant for the other.

Assumption (Principle of Common Cause (PCC))

If two event types A and B are correlated and the correlation cannot be explained by direct causation or by event identity, there there is a common cause variable s.t. $p(A \wedge B | V) = p(A|V)\, p(B|V)$.

SEPARABILITY, LOCALITY 1, and PCC jointly entail (together with auxiliary assumptions) $p(L_i^a \wedge R_j^b | V) = p(L_i^a | V)\, p(R_j^b | V)$.

Assumption (Locality 2)

If $L_i \wedge R_i \wedge X$ is sufficient for L_i^a, then $L_i \wedge X$ is alone sufficient for L_i^a. Similarly, if $L_j \wedge R_j \wedge Y$ is sufficient for R_j^b, then $R_j \wedge Y$ alone is sufficient for R_j^b.

Assumption (No conspiracy)

The common cause variable V is not influenced by the setting or the measurement operations in the two wings: $p(V | L_i \wedge R_j) = p(V)$.

The result on top of this page, together with LOCALITY 2 and NO CONSPIRACY (and auxiliary assumptions) then yields the Bell inequality (although I have glossed over some subtleties).

Upshot

So Bell locality must be violated. But since the assumption of Bell locality can be unpacked into several weaker assumptions, there are various ways in which it can be violated:

- The measurement events in the two wings are not separate.
- One of the measurement events instantaneously causes the other.
- There is no common cause at the source.
- The settings in one wing have a causal influence on the measurement in the other wing.
- There is backward causation such that the settings in either or both of the wings (which can be set after the particles departed the source) causally influence the common cause at the source event.

Note: one of these must be true.

Additional considerations on nonlocality

Albert, *Quantum Mechanics and Experience*, Ch. 3.

- EPR thought that the nonlocal character of measurements on non-separable states is a merely disposable artifact of the particular formalism of standard QM.
- The upshot of Bell's theorem is that this is demonstrably wrong:

"What Bell has given us is a proof that there is as a matter of fact a genuine nonlocality in the actual workings of nature, however we attempt to describe it, period. That nonlocality is... necessarily... a feature of every possible manner of calculating... which produces the same statistical predictions as quantum mechanics does; and those predictions are now experimentally known to be correct." (70)

Nonlocality is subtle

- Given the non-separable state $|\psi\rangle$, the statistics of outcomes of spin measurements on electron in *L*-wing depend nonlocally on the outcomes of spin measurements on electron in *R*-wing, and vice versa.
- But do the statistics of the outcomes of spin measurements on an *L*-electron, given $|\psi\rangle$, depend nonlocally on whether a spin measurement is performed on the *R*-electron, or vice versa?
- Given $|\psi\rangle$, the outcome of a measurement of the colour of the *L*-electron is equally likely 'white' or 'black', whether or not a measurement of the colour of the *R*-electron has previously been made. (Why?)
- In fact, since $|\psi\rangle = 1/\sqrt{2}(|^1\text{hard}, ^2\text{soft}\rangle - |^1\text{soft}, ^2\text{hard}\rangle)$, the outcome of a colour measurement of the *L*-electron is equally likely 'black' or 'white', regardless of whether a hardness measurement is first carried out on the *R*-electron. (Why?)

In fact, we have the following general result:

Theorem

For any state $|Q\rangle$ of a quantum system S consisting of two subsystems s_1 and s_2, and any observables $\hat{A}$ of s_1 and $\hat{B}$ of s_2, the probabilities of the various possible outcomes of a measurement of $\hat{A}$ don't depend on whether or not a measurement of $\hat{B}$ is carried out first. (72)

- outcomes of measurements sometimes depend nonlocally on outcomes of other (distant) measurement, but they don't depend nonlocally on whether or not any other (distant) measurements get carried out

⇒ nonlocality cannot be exploited to transmit detectable signals between distant locations

Three final comments

Tim Maudlin, *Quantum Non-Locality and Relativity*, Ch. 1.

I will not go over all in the chapter since it duplicates a lot of what I said so far already three times. But you should have a good look at the thought experiment in the section 'How do they do it?'.

Three results concerning the 'quantum connection':

1. It is unattenuated: in contrast to classical (instantaneous) action, the quantum connection is unaffected by distance.
2. It is discriminating: while gravitational forces affect similarly situated objects in the same way, the quantum connection is a private arrangement between entangled particles.
3. It is instantaneous: while Newton's theory of gravity has gravity propagate instantaneously, it need not do so, and GR certainly involves no instantaneous gravitational action; but the quantum connection appears to act essentially instantaneously.