Collapse

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MA Seminar: Philosophy of Physics

Collapse of the wave function

"If one has to stick to this damned quantum jumping, then I regret having ever been involving in this thing." (Jammer 1974: 344, ref.4, Pais 1988: 261)

The idea of a collapse

Albert, Ch. 5.

- We have already seen Dirac's argument that there must be a collapse of the wave function (cf. Barrett's text).
- John von Neumann, *Die mathematischen Grundlagen der Quantenmechanik* (1932)
- For the same reasons as Dirac, Neumann argued that there must two dynamical regimes:
  1. no measurements: dynamical laws of motion, Schrödinger equation
  2. when there are measurements occurring: dynamical evolution according to collapse postulate, not in accordance with dynamical laws
- Problem: what this really amounts to depends on exact meaning of term 'measurement'...

- Suppose we have a quantum state in an eigenstate of operator $\hat{A}$ and we make a measurement of $\hat{B}$ which doesn't commute with $\hat{A}$.
- We know: by the time the measurement is done (a sentient observer has formed a belief about what the measurement device indicates), there must have been a violation of dynamical law, i.e. a collapse
- Precisely when does the collapse occur?
- First attempt (by Wigner): always happens precisely at the level of consciousness
- Note that this proposal implies (substance) dualism.
- Problem: what precisely is conscious and what isn't?

- Second attempt: microscopicness (i.e. collapse occurs at the last ‘reasonable’ moment)
- Idea: whenever two macroscopically distinct states get superposed, the state collapses
- This proposal implies that there are two sorts of physical systems: (1) purely microscopic systems (those which don’t contain macroscopic subsystems) that always evolve in accordance with the dynamical laws so long as there is no interaction with macroscopic systems; and (2) macroscopic systems that evolve according to more complicated laws of motion
- Problem: what precisely is ‘macroscopic’

Couldn't we settle the issue empirically?

- Couldn't we observe precisely where and when the collapse (a physical event!) occurs?
- Consider the standard measurement setup: a black e^- is measured by a hardness box.
- Two theories: one, T_1, entails that the collapse occurs when the measuring device becomes correlated with the e^-, the other, T_2, that collapse occurs at some later moment in time, e.g. when the light from the pointer reaches the retina of the observer
- More precisely: there is a moment s.t. according to T_1, the state of the measuring device-cum-e^- is

$$\begin{array}{lll} \textit{either} & |\text{`hard'}\rangle_m |\text{hard}\rangle_e & \text{(with probability 0.5)} \\ \textit{or} & |\text{`soft'}\rangle_m |\text{soft}\rangle_e & \text{(with probability 0.5)} \end{array} \quad (1)$$

- According to T_2, however, the state of the total system at that moment ought to be

$$\frac{1}{\sqrt{2}}|\text{`hard'}\rangle_m|\text{hard}\rangle_e + \frac{1}{\sqrt{2}}|\text{`soft'}\rangle_m|\text{soft}\rangle_e \qquad (2)$$

- The problem now becomes one of figuring out whether we can observationally distinguish between these two states, i.e. whether there is an observable that would mark this difference.
- What about measuring the position of the tip of the pointer?
- For T_1, the pointer would have a determinate position (each with chance of 50%), while for T_2, this measurement has a fifty-fifty chance each of collapsing the state of the pointer onto 'hard' or onto 'soft'.

$\Rightarrow$ probabilities of outcomes will be exactly the same

- measuring the hardness or the colour of the e^- won't work either as the probabilities will be exactly the same (Why?)

- Consider an observable that Albert dubs 'zip' (by property (5) of Hermitian operators, there is necessarily such an observable), wtih the following eigenstates:

$$
\begin{aligned}
|\text{zip} = 0\rangle &= |\text{ready}\rangle_m \\
|\text{zip} = +1\rangle &= \frac{1}{\sqrt{2}}\left(|\text{`hard'}\rangle_m + |\text{`soft'}\rangle_m\right) \\
|\text{zip} = -1\rangle &= \frac{1}{\sqrt{2}}\left(|\text{`hard'}\rangle_m - |\text{`soft'}\rangle_m\right)
\end{aligned} \tag{3}
$$

- Measuring zip won't work, since if either the state in (1) or (2) obtains, this measurement has both a fifty-fifty chance of collapsing the state of pointer onto $|\text{zip} = +1\rangle$ and onto $|\text{zip} = -1\rangle$.
- This could have been directly seen: state (2) is nonseparable state of e^- and device, a state in which neither the e^- nor the device alone can have any definite properties.

- Consider the observable zip − color.
- State in (2) is eigenstate of zip − color (with eigenvalue 0), and neither of states in (1) is.

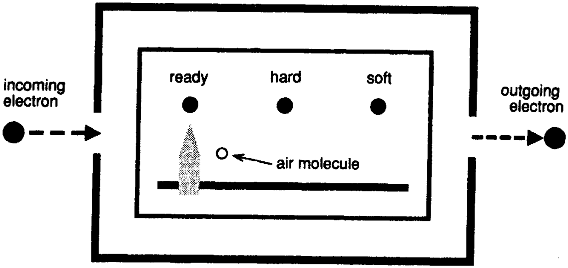
⇒ on T_1, when (1) obtains, we get a statistical distribution of different outcomes, but on T_2, when (2) obtains, we get, by necessity, outcome 0

⇒ there exists in principle a measurement that could distinguish between T_1 and T_2

- But this measurement turns out to be extremely difficult to perform...

Interaction of the measuring device with the environment

- Due to their 'macroscopicness', measuring devices interact with their environment.
- Suppose that an incoming black e^- is fed into the 'ready' hardness box, and suppose that there is an molecule of air just to the right of the pointer s.t. if the pointer swings to 'hard' position, the molecule is pushed to the center of the dial and if the pointer swings to the 'soft' position, it's pushed to the right end of the dial.



- The state of the system e$^-$ and hardness box and air molecule after the e$^-$ has moved through the box will be, on T_2,

$$\frac{1}{\sqrt{2}}\left(|\text{hard}\rangle_e|\text{`hard'}\rangle_m|\text{center}\rangle_a + |\text{soft}\rangle_e|\text{`soft'}\rangle_m|\text{right}\rangle_a\right) \tag{4}$$

⇒ This is not an eigenstate of zip − color anymore...

- On the other hand, on T$_1$, the total state is

$$\begin{array}{ll} |\text{hard}\rangle_e|\text{`hard'}\rangle_m|\text{center}\rangle_a & \text{(with probability 0.5)} \\ |\text{soft}\rangle_e|\text{`soft'}\rangle_m|\text{right}\rangle_a & \text{(with probability 0.5)} \end{array} \tag{5}$$

(neither of which is an eigenstate of zip − color).

- Unlike (1) and (2), (5) and (4) cannot be distinguished.
- Problem: state in (4) is nonseparable between e$^-$ and box and air molecule

- There exists an observable that would distinguish (5) and (4), but that would necessarily be more complicated than zip $-$ color, as it must be an observable of the composite system of all three objects...
- And there will be more air molecules, or photons, or tiny specks of dust...

$\Rightarrow$ "all of them together... will fantastically increase the complexity of the observables which need to be measured... in order to distinguish between [T_1 and T_2]... The upshot of all this... is that different conjectures about precisely where and precisely when collapses occur are the sorts of conjectures which (for all practical purposes...) cannot be empirically distinguished from one another." (91)

Conditions on a theory of collapse

Condition (i: Measurements have outcomes)

"We want [a theory of collapse] to guarantee that measurements... always have outcomes; we want it to guarantee... that there can never be any such thing in the world as a superposition of 'measuring that A is true' and 'measuring that B is true.' " (92)

Condition (ii: Statistical connection)

"We want it to preserve the familiar statistical connections between the outcomes of those measurements and the wave functions of the measured systems just before those measurements. That is, we want it to entail, or we want it at least to be consistent with, principle D." (93)

Condition (iii: Empirically correct dynamics)

"We want it to be consistent with everything which is experimentally known to be true of the dynamics of physical systems. We want it, for example, to be consistent with the fact that isolated microscopic physical systems have never yet been observed not to behave in accordance with the linear dynamical equations of motion, the fact that such systems, in other words, have never yet been observed to undergo collapses." (ibid.)

Ghirardi-Rimini-Weber (1986): the GRW theory

- search for physical mechanism of collapse
- outcomes are typically recorded in the position of something (e.g. tips of pointers, drops of ink on paper, etc.)

⇒ Can we construct a theory which entails that every macroscopic object always has some particular position?

- Suppose that there exists, by natural law, a small, fixed probability for each quantum system independently that it will collapse during a unit time interval s.t. the position onto which it collapses is probabilistically determined by postulate D.
- Consider a particle in state

$$|A\rangle = a_1|x_1\rangle + a_2|x_2\rangle + a_3|x_3\rangle + \ldots \tag{6}$$

⇒ Probability that collapse will leave particle in state $|x_i\rangle$ is $|a_i|^2$

- If collapse of particle in state (6) occurs, then one of the terms in (6) gets multiplied by a finite number, all others by 0.
- After collapse, particle will evolve according to Schrödinger eq.
- Probability that the ith term in (6) is the one that gets multiplied upon a collapse is $|a_i|^2$, and the number by which it gets multiplied is $1/a_i$. (Why?)

"What happens when a particle undergoes a collapse is that the wave function of the particle gets multiplied by an eigenfunction of the position operator... and the probability that the position eigenvalue of that position eigenfunction is x_i is stipulated to be equal to $|\langle x_i|w\rangle|^2$ (where $|w\rangle$ is the state of the particle at the moment just before the collapse occurs); and note that the outcome of this multiplication (that is: the product of these two wave functions), whatever $|w\rangle$ happens to be, is invariably also an eigenfunction of the position operator with eigenvalue x_i." (95)

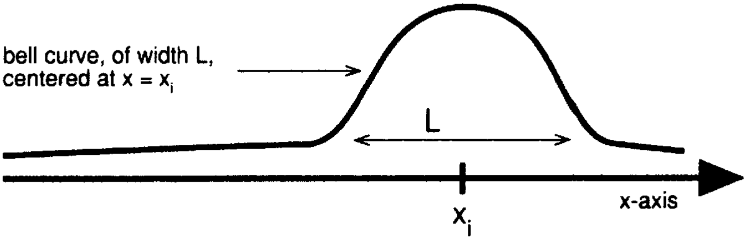
- Consider what happens in macroscopic systems, such as pointer in hardness box. This pointer consists of trillions of particles (labelled by numbers).
- The state in (2) then is (written out in terms of the states of those particles)

$$\frac{1}{\sqrt{2}}\left[(|x_1\rangle_1|x_1\rangle_2|x_1\rangle_3\cdots)|\text{hard}\rangle_e + (|x_2\rangle_1|x_2\rangle_2|x_2\rangle_3\cdots)|\text{soft}\rangle_e\right]$$

  where x_1 (x_2) is the pointer position pointing to 'hard' ('soft').
- Suppose one of these particles now undergoes collapse (the probability for this will be high since the pointer consists of a very large number of particles).
- Suppose, e.g., that the *i*th particle will be collapsed into $|x_1\rangle_i$.

- But that means that the entire second term of the superposition state on the previous slide vanishes, and the state will turn into the first state in (1)!
- GRW theory thus entails that collapses almost never happen to isolated microscopic systems, but that states consisting of many, many particles, collapse almost certainly and almost immediately with the standard quantum mechanical probabilities (e.g. into one of the states of (1)).
- But what about uncertainty? If position precise, momentum maximally uncertain...
- But the GRW theory adds a slight modification that takes care of this:

"Stipulate that when a particle undergoes a collapse, what its wave function gets multiplied by isn't an eigenstate of the position operator but is rather a bell-shaped function like the one in figure 5.3.



"Also stipulate that the probability of that bell curve's being centered at the point x_i (if such a collapse happens to occur) is proportional to to $|\langle B_i|w\rangle|^2$ (where $|B_i\rangle$ is the bell-curve state centered at x_i and $|w\rangle$ is the state of the particle just prior to the collapse... What we want from [collapses...] is to insure that macroscopic objects... almost invariably have almost determinate locations. And it turns out that this revised prescription can deliver that; it turns out that the bell curves can be made narrow enough so that whatever uncertainties there are in the positions of macroscopic things are almost invariably microscopic ones. And it turns out... that these curves can nonetheless be made wide enough... so that the violations of the conservation of energy and or momentum which the multiplication by these curves will produce will be too small to be observed." (97f)

Problems for the GRW theory

- So is everything fine? Not quite...
- Trouble: bell-curved functions are nonzero everywhere (although very small)

$\Rightarrow$ Strictly speaking: particles are still in superposition states of being all over the place and thus don't put anything even in approximately determinate locations.

$\Rightarrow$ revised prescription cannot insure that measuring devices with pointers ever have definite outcomes...

- What needs to be explained is why to put the state vectors of pointers close to those with definite outcomes suffices for all relevant purposes.
- But let's press on.

Experiments with television screens

- Suppose that GRW thus satisfies the requirements for a collapse theory as expressed by the three conditions, at least in those cases where measurement outcomes are indicated by spatial positions of pointers of ink dots or something.
- Problem: not all measuring devices work like that: television screens that get illuminated to indicate that they detected an incoming e^- in a certain position operate not by changing spatial position, but by changing energetic states of its atoms (or their e^-).

"Here's the crucial point: the GRW 'collapses' are invariably collapses onto (nearly) eigenstates of position, but it's the energies of the fluorescent electrons, and not their positions, that get correlated, here, to the hardness of [the incoming particle]! The GRW collapses aren't the right sorts of collapses to precipitate an outcome of the hardness measurement here." (101)

"It turns out... that there can be genuinely macroscopic measuring instruments that (nonetheless) have absolutely no macroscopic moving parts. That's what's been overlooked in the GRW proposal. What the GRW theory requires in order to produce an outcome [is...] that the recording process involve macroscopic changes in the position of something." (103f)