Arguments, deduction, induction

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Singular ropositions

Definition

A singular proposition is a proposition concerning an individual.

Exemples

(1) Galileo observed three moons of Jupiter in Padua on 7 January 1610.

(2) This litmus paper turns red when immersed in the liquid.

(3) At this instant, the temperature on the peak of Mont Blanc is below zero.

(4) A Higgs boson has been observed at CERN in 2012.

Universal propositions

Definition

A universal proposition is a proposition concerning all elements of a class.

Exemples

(1) All cetaceans have lungs.

(2) The planets revolve around their suns on elliptical orbits.

(3) When a ray of light passes from one medium to another, the direction of the refracted ray is such that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant characteristic of the two media (= law of refraction).

(4) The acid turns litmus paper red.

Observation statements

Definition

An observation statement is the expression, in a language, of the result of an empirical observation.

Exemples

(1) This liquid turns the litmus paper red.

(2) The deflection of the pointer is 21 degrees.

(3) The ball rolled 2 metres in 1.2 seconds.

Theoretical statements

Definition

A theoretical statement is the expression, in a language, of the facts put forward by a theory, often concerning the relationship between theoretical entities.

Exemples

(1) The current is 2 Ampères.

(2) Any change in the intensity of the electric field induces a magnetic field.

(3) Genes contain the information needed to synthesise proteins.

‘Theory-ladenness’: all experience is pervaded by theory

- According to some philosophers of science (William Whewell, Thomas Kuhn, Paul Feyerabend), all the statements we accept based on perceptions or experiences, depend on our theoretical assumptions.
- An observation statement is always formulated in the language of a pre-existing theory. They are therefore not purely empirical statements.

⇒ Thus, the traditional dichotomy between observational and theoretical statements is problematic.

- Nevertheless, we can distinguish between different kinds of statements that play different roles in scientific knowledge.

General characteristics of deduction and induction

Definition (Argument)

An argument is a set of propositions, some of which (the 'premises') serve to justify another proposition (the 'conclusion') in the set.

- An argument provides reasons for believing something.
- Arguments can be either deductive or inductive.

Deduction: definitions

Definition (Deduction)

A deduction is a form of argument which joins a conclusion to the premises by means of a (logically) valid inference.

Definition (Validity)

An inference is valid if and only if (iff) it is impossible for the conclusion to be false if the premises are true.

Deduction: remarks

- Deductions are characterised by the impossibility to combine premises and conclusions such that all premises are true and the conclusion is false.
- Test: can we imagine a situation in which the premises are all true but the conclusion is false?
  - ⇒ If this is possible, then the argument is not valid.
- Logical impossibility: in virtue of the logical form of the argument

Deduction: examples

Example of a deduction: ‘syllogism’

(p_1) All mammals have lungs.

(p_2) All cetaceans are mammals.

(c) Thus, all cetaceans have lungs.

Deduction: examples

Substitution:

(p_1) All Ms are Ps.

(p_2) All Cs are Ms.

(c) Thus, all Cs are Ps.

Deduction: examples

Validity and truth:

(p_1) All fish have lungs.

(p_2) All cetaceans are fish.

(c) Thus, all cetaceans have lungs.

Deduction: examples

Validity and truth:

(p_1) All fish have legs.

(p_2) All cetaceans are fish.

(c) Thus, all cetaceans have legs.

$\Rightarrow$ valid argument!

Deduction: examples

Validity and truth:

(p_1) All mammals have lungs.

(p_2) All cetaceans have lungs.

(c) Thus, all cetaceans are mammals.

$\Rightarrow$ invalid argument!

Deduction: lesson

- Lesson: we must distinguish the truth of the premises, or even of the conclusion, from the validity of the argument!
- We distinguish 'soundness' from 'validity'. A sound argument is a valid argument of which all premises are true.

Warning

Propositions are true or false, but never valid or invalid; in constrast, arguments are valid or invalid, but never true or false.

Deduction: typical form

On of the premises is a universal propositions, the conclusion is a singular proposition:

(p_1) All men are mortal.

(p_2) Socrates is a man.

(c) Thus, Socrates is mortal.

Non-ampliativity

(p_1) All mammals have lungs.
(p_2) All cetaceans are mammals.
(c) Thus, all cetaceans have lungs.

This valid inference is not ampliative: it does not add any supplementary contents to the premises. If you already know that all mammals have lungs and that all cetaceans are mammals, then you already know (perhaps implicitly) that all cetaceans have lungs.

Inductive inference: characterisation

Characterisation (Induction)

An induction is a form of argument that infers a more general and sometimes universal conclusion from premises that express more specific facts.

- An inductive inference is ampliative.
- An inductive inference is not valid. (Why?)

Inductive inference: ampliativity

(p) The Sun has risen every morning until now.
(c) Thus, the Sun will rise on the morning of 20 September 2193.

This inference is ampliative: the statement that the Sun rises every morning until now does not 'contain' the statement that it will rise on 20 September 2193. Nevertheless, it appears to be a 'good' inductive inference, even if it goes beyond the content of the premise.

But in virtue of what is it a 'good' inference?

Example: Boyle's law

Boyle's law

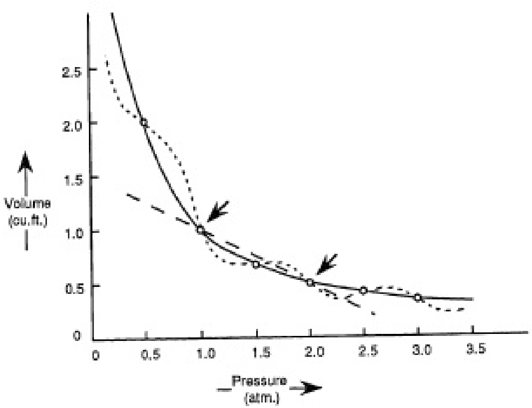
For a given quantity of any ideal gas at a given temperature,

$$PV = \text{const},$$

where P is the pressure and V is the volume of the gas.

- The inference to this law from the measurement data is also ampliative since the measured data does not contain the law.
- The inference to Boyle's law is not unique: one of the conclusions consistent with the data is selected.

Example: Boyle's law



In virtue of what is the inference to Boyle's law a 'good' or even the 'best' possible inference?

Material inference

Substitution:

(p) I have waken up every morning until now.

(c) Thus, I will wake up on the morning of 20 September 2193.

This inference is not ‘good’, even though it has exactly the same form!

Material inference

(*a*) It has rained, the streets are thus wet.

- The quality of the inference depends on the content of the two propositions in (a). It is thus a material inference.
- Contrast this to deductive inferences:

(p_1) All snarks are blert.
(p_2) Henry is a snark.
(c) Thus, Henry is blert.

$\Rightarrow$ The validity of this inference is independent of the content of the involved propositions. It is a purely formal matter.

David Hume: the problem of induction

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David Hume (1748). *An Enquiry Concerning Human Understanding* (Section V).

Sam Dresser: I hope this helps: The problem of induction

https://www.youtube.com/watch?v=Fd1U_MC_p3M

- Hume distinguishes between relations of ideas and facts.
- relations of ideas: can be known independently of experience, concern the abstract space of logic and mathematics, all knowledge which is analytic *a priori*
- facts: everything other than relations of ideas, concerns material existence, all knowledge which is synthetic
- Facts can be observable (e.g. 'There is a table here'), or unobservable (e.g. 'The Sun will rise tomorrow').
- In order to know a fact beyond what is directly given to us by experience, we are obliged to use inductive reasoning.

David Hume: the problem of induction

- However, inductive inferences depend on a 'principle of uniformity of nature': the past is a faithful guide to the future.
- According to Hume, such a principle cannot be justified. Its rational justification, if there is one, could take two different forms:
    1. Demonstrative, deductive reasoning *a priori*; however, the future does not logically depend on the past because it is conceivable that the future does not resemble the past; thus, we cannot base induction on reasoning *a priori*.
    2. Inductive reasoning: our past success using inductive inferences guarantees our inductive inferences towards the future; circular!

Conclusion

Our inductive practices cannot have a rational foundation.

Deduction and induction

Deductive inferences:

- non-ampliative
- infallible
- formal

Inductive inferences:

- ampliative
- fallible
- material

Necessary connection between fallibility and ampliativity

If an inference brings in new content, this is a potential source of error. This is why deductive inference must be non-ampliative. In contrast, it is the very purpose of inductive inference to generate new content, so it must be ampliative. But this ampliativity is not 'free' as it were; it comes with the price of being fallible.

Types of induction and their problems

(a) Enumerative induction (and projection)
(b) Eliminative induction
(c) Causal inference
(d) Inference to the best explanation (explanatory inference, abduction)

(a) Enumerative induction

(1) x_1 is F and is also G.
(2) x_2 is F and is also G.
$\vdots$
(n) x_n is F and is also G.
$\therefore$ All xs that are F are also G. $[\forall x(Fx \rightarrow Gx)]$

- Projection: an inference from observed cases to the next case of that type (not to all cases)
- Complete induction: if there exist no other xs (a type of deduction, and not of induction despite its name; employed in mathematics)

(a) Enumerative induction

- Enumerative induction is somewhat too simple to play much of a role in science, but there are some examples, e.g.:

Example: Chargaff's rules

- Erwin Chargaff (1951): Among the four bases of DNA—C, A, T, and G—, the amounts of C and G are always (roughly) the same, as are the amounts of A and T.
- Chargaff found this by enumerative induction in the late 1940s, based on a small number of cases (including, however, a diverse range or organisms).

(b) Eliminative induction

In case we have several hypotheses $h_1, h_2, \ldots$ that lead to different predictions:

(1) $h_1 \rightarrow e$
(2) $h_2 \rightarrow e$
(3) $h_3 \rightarrow \neg e$
(4) $\neg e$
$\therefore$ h_3

Elimination of h_1 and h_2 by modus tollens (deduction):

(1) $p \rightarrow q$
(2) $\neg q$
$\therefore$ $\neg p$

- Eliminative induction transforms into a kind of deduction if we add the disjunctive premise $h_1 \vee h_2 \vee h_3$.
- In the empirical sciences, it is normally impossible to prove such an exhaustive disjunction. This is why Pierre Duhem argued that crucial experiments were impossible.

Definition (Crucial experiment (experimentum crucis))

A crucial experiment is an experiment that conclusively refutes one of two competing hypotheses (or theories), thereby establishing the rival thesis or theory.

(We will return to crucial experiments in the module on underdetermination.)

Pierre Duhem (1861-1916)

The Aim and Structure of Physical Theory (1906, 188-190)

> *Do you wish to obtain from a group of phenomena a theoretically certain and indisputable explanation? Enumerate all the hypotheses that can be made to account for this group of phenomena; then, by experimental contradiction eliminate all except one; the latter will no longer be a hypothesis, but will become a certainty...*
>
> *Between two mutually theorems in Geometry there is no room for a third judgment; if one is false, the other is necessarily true. Do two hypotheses in physics ever constitute such a strict dilemma?*

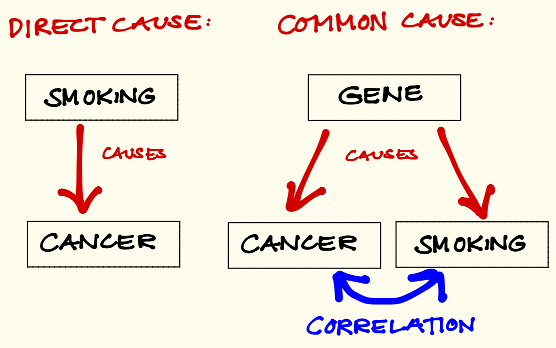
Pierre Duhem (1861-1916)

The Aim and Structure of Physical Theory (1906, 288f)

> *Shall we ever dare to assert that no other hypothesis is imaginable? Light can be a swarm of projectiles, or it may be a vibratory motion whose waves are propagated in a medium; is it forbidden to be anything else at all?*
>
> *Unlike the reduction to absurdity employed by geometers, experimental contradiction does not have the same power to transform a physical hypothesis into an indisputable truth; in order to confer this power on it, it would be necessary to enumerate completely the various hypotheses which may cover a determinate group of phenomena; but the physicist is never sure he has exhausted all the imaginable assumptions. The truth of a physical theory is not decided by heads or tails.*

(c) Causal inference

- Example: observing a correlation between smoking and lung cancer, how can we know that it is the smoking which causes the lung cancer?
- Two causal models:



- To infer the existence of causal relations from simple correlations is the central problem of causal inferences.

John Stuart Mill (1806-1873)

A System of Logic: Ratiocinative and Inductive, 1843

- Mill offers five methods of causal inference:
  1. The method of agreement
  2. The method of difference
  3. The joint method of agreement and difference
  4. The method of residues
  5. The method of concomitant variations
- All methods start by identifying variables assumed to include the possibles causes. Then they use correlations to separate actual causes from possible causes. (All this was prior to the development of statistical methods.)

(1) The method of agreement

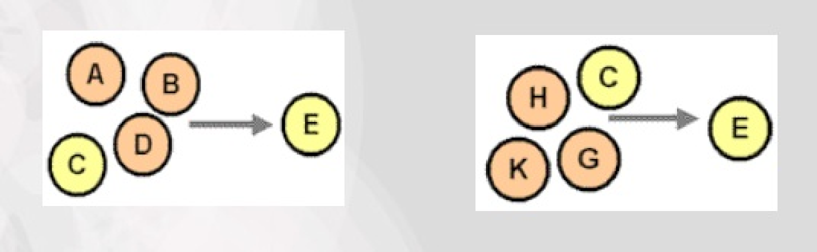
A System of Logic, Vol. 1, (1843, 454)

> *«If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon.»*

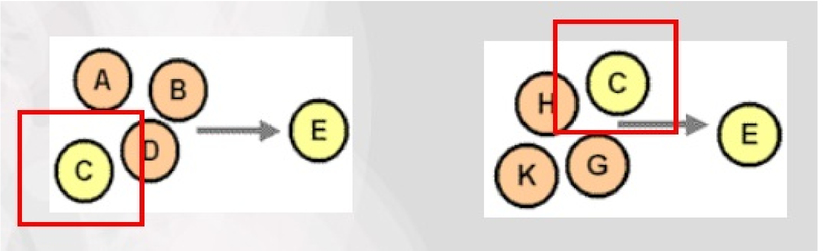
Method (Agreement)

1. *Find cases in which the effect has occurred*
2. *Determine if there is only one thing that they all share*
3. *If there is, that is (the likely) cause*

(1) Method of agreement



(1) Method of agreement



(1) Method of agreement: example

Example:

- It turns out that some cities have markedly lower rates of tooth decay.

⇒ If fluoride in the water is the only (potentially relevant) thing in common, then it is the likely cause.

	Dental education program	Free dental clinics	Fluoride in water	High Salaries for dentists	Low rates of tooth decay
Donaldville	Yes	No	Yes	No	Yes
Duckburg	No	Yes	Yes	Yes	Yes
Entenhausen	No	No	Yes	No	Yes
Patoburgo	Yes	Yes	Yes	No	Yes

(1) Method of agreement: example

Example:

Five patients all show amnesia after brain injury:

- Patient 1: damage to the prefrontal cortex and the hippocampus
- Patient 2: damage to the hippocampus, amygdala, and entorhinal cortex
- Patient 3: damage to the thalamus and hippocampus
- Patient 4: damage to the prefrontal cortex, hypothalamus, hippocampus, and amygdala
- Patient 5: damage to the hippocampus and amygdala

(1) Method of agreement: example

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(2) Method of difference

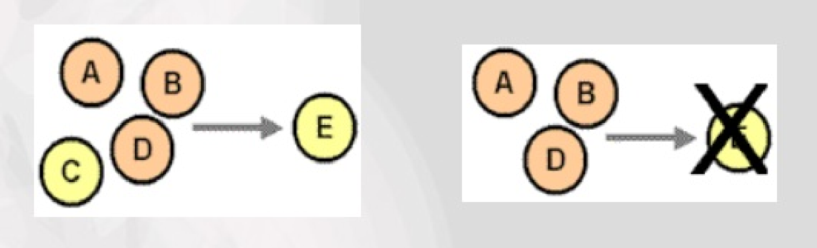
A System of Logic, Vol. 1, (1843, 430)

«If an instance in which the phenomena under investigation occurs and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former, the circumstance in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.»

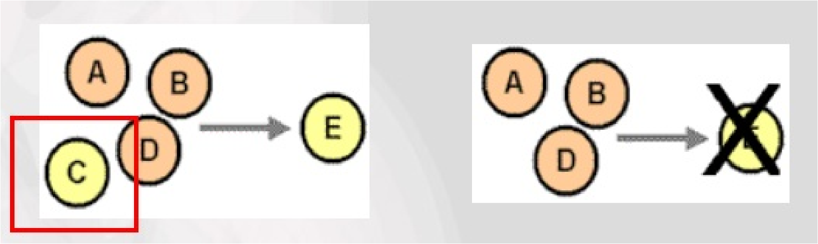
Method (Difference)

1. *Find two things that differ in that one has the effect and the other doesn't.*
2. *If there is only one factor on which they differ, that is the likely cause.*

(2) Method of difference



(2) Method of difference



(2) Method of difference: example

Example:

- Four people apply for a loan, but only two get it.
- The only difference is that those who were denied once declared bankruptcy.

⇒ The declaration of bankruptcy is the likely cause of the loan being turned down.

	College education	Earn over CHF 100K	Own business	Declared bankrupcy	Loan approved
Etienne	Yes	Yes	No	Yes	No
Juliette	Yes	Yes	No	No	Yes
Rémi	Yes	Yes	No	No	Yes
Valérie	Yes	Yes	No	Yes	No

(2) Method of difference: the example of yellow fever

In 1900, Major Walter Reed was given the responsibility of finding the cause of yellow fever and eliminating it. After many unsuccessful experiments, he decided to test an old but unproven theory that the disease was transmitted by mosquitos. Unfortunately, no animal was known to be susceptible to yellow fever at the time, so it was necessary to use human volunteers. In the painting, Dr. Lazear, who died a month later as a result of self-experimentation, is shown inoculating Dr. Carroll with an infected mosquito, The experiment proved conclusively that the mosquito was the carrier of yellow fever.

(2) Method of difference: the example of yellow fever

(2) Method of difference: the example of yellow fever

Once Walter Reed suspected mosquitoes as the transmitter of yellow fever,

- he had one set of volunteers sleep on the soiled clothes and beds of yellow fever patients in a room screened so that no mosquitoes could get in.
  - $\Rightarrow$ None of these people contracted the disease.
- he had another group of volunteers stay completely away from sick patients, except he let mosquitoes that had been allowed to feast first on people sick with the disease bite the patients.
  - $\Rightarrow$ These volunteers did get sick.

(2) Method of difference

Another example

In order to determine the efficacy of a pharmacological substance, two very different groups are compared: one receiving the substance and the other a placebo.

Critical questions

- Are Mill's methods absolutely faithful?
- Under which conditions do they work?
- How can we control the risk of confounding factors?
- Some factors may seem to be a cause, even though they are in fact only an effect of a different common cause.

(d) Inference to the best explanation (IBE)

This inference is also called ‘explanatory inference’ or ‘abduction’.

The methodological rule IBE:

Given a set of alternative hypotheses, we need to choose the one that provides the best explanation of the phenomena in question.

Example: Dr. House and the method of differential diagnosis

- A patient presents a series of symptoms. House and his team compose a list of diseases for which these symptoms are typical. For example, lupus, pericarditis, lung cancer, etc.
- House requests further tests (= additional symptoms)
- Then, Houses chooses the disease which best explains all the symptoms (including the test results), and exclusively the present symptoms.
- This is the best explanation of the symptoms.

Explanations vs. potential explanations

Peter Lipton, *Inference to the Best Explanation*, 2nd ed. 2004

- Peter Lipton: We need to distinguish between explanations and potential explanations. A potential explanation makes a fact or phenomenon comprehensible. An explanation is a potential explanation that is also true.
- We also need to distinguish between the ‘likeliest explanation’ and the ‘loveliest explanation’.
- The likeliest explanation: the explanation which is most probable
- The loveliest explanation: the explanation which is most explanatory, gives us most understanding.

- According to Lipton, the inference rule IBE must be reconstructed as inference to the loveliest potential explanation
- If we assumed that the rule IBE infers to the likeliest explanation, it would be like a dessert recipe that starts with "make a chocolate soufflé".
- The central idea of IBE is that we use the 'loveliness' as an indicator of 'likeliness' (and thus of truth).

Problems of IBE

1. Is 'understanding' really a reliable guide to the truth or an indicator of the probability of hypotheses?
2. Bas van Fraassen (*The Scientific Image*, 1980): 'best of a bad lot' objection

$\Rightarrow$ The best explanation may always be the best of a bad lot. Even if it is possible to classify a set of hypotheses according to their explanatory value, we do not really know whether the best hypothesis is true.