Intro to Philosophy of Mathematics Homework 1

History of philosophy of mathematics, set theory Due: 21 March 2022

1. Shapiro characterizes Plato as defending a 'double realism' concerning mathematics. Explain what he means by that. [1 point]

2. Hilbert's Hotel, as it turns out, it just one hotel of *Hilbert's Chain* of infinitely many hotels with infinitely many rooms each. Suppose that all hotels of Hilbert's Chain are fully booked with exactly one guest in each room of each hotel. One day, the management decides to remodel all but one of the chain's hotels. In order to loose no business, they hatch a plan of how to accommodate all the guests in all the hotels which close down in the one hotel which will remain open during the remodelling of the others.

   (a) Is this possible? If so, how? If not, why not? [2 points]

   (b) Which of the two infinities—the Chain's total number of rooms prior to and during the remodelling—, if any, is larger? [1 point]

3. Alice and Bob play the following game together. Alice writes down a finite set of natural numbers. Bob does not know either *how many* numbers she wrote down or which is the largest number among those on the paper. Is there a strategy for Bob to solve this problem in finitely many steps? If so, how can we denumerate the set of finite sets of natural numbers? If not, why not? [2 points]

4. Zermelo accepted the following axiom of set theory:

   Axiom (Axiom schema of specification)**.** *Given an arbitrary property as well as an arbitrary set S, then the set of all elements of the set S which exemplify the property exists.*

   Show that the universal set U cannot exist under the assumption of this axiom. This resolves Cantor's paradox of the universal set. [2 points]

5. An *asymmetric relation* R is a binary relation defined on a domain S such that $(\forall a, b \in S, Rab \rightarrow \neg Rba)$. Show that the asymmetry of a relation entails that it is irreflexive, i.e. $\forall a \in S, \neg Raa$. [2 points]

6. For extra credit, do Exercise 28 on the handout on set theory. [Automatic 10 for the course]