Intro to Philosophy of Mathematics Homework 2

Logicism, formalism Due: 4 April 2022

1. What is an impredicative definition? Why might one think that impredicative definitions are problematic? [1 point]

2. How does Russell define natural numbers? How does he define rational numbers? How does this force him to assume an axiom of 'infinity'? [2 points]

3. What is the declared goal of logicism? To what extent did it succeed in this ambition? To what extent did it fail? [3 points]

4. A term formalist identifies mathematical entities with their names. What is Frege's main objection to term formalism? Does the objection succeed? What could a term formalist say in response to Frege? [2 points]

5. How do Gödel's incompleteness theorems affect Hilbert's program? As a reminder, here are informal statements of Gödel's two theorems (no need to repeat them in your answer):

   Theorem 1 (Gödel's first incompleteness theorem)**.** *Let T be a formal system. If T contains some arithmetic, and it can be determined algorithmically whether a given sequence of characters is a well-formed formula and a valid deduction in T, then "there is a sentence G in the language of T such that*

   (a) if T is consistent, then G is not a theorem of T, and

   (b) if T has a property a bit stronger than consistency... then the negation of G is not a theorem of T." (Shapiro, 166)

   Theorem 2 (Gödel's second incompleteness theorem)**.** *"[I]f the formalization of 'provable in T' meets some straightforward requirements, then we can derive, in T, a sentence that expresses the following:*

   If T is consistent, then G is not derivable in T.

   But, as noted above, 'G is not derivable in T' is equivalent to G. So, we can derive, in T, a sentence to the effect that

   If T is consistent then G.

   Assume that T is consistent, and that we can derive, in T, the requisite statement that T is consistent; then it would follow that we can derive G in T. This contradicts the [first] incompleteness theorem. So if T is consistent, then one cannot derive in T the requisite statement that T is consistent." (166f)

   [2 points]