Intro to Philosophy of Mathematics Homework 3

Transfinite mathematics, intuitionism Due: 11 April 2022

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1. Prove that the Sets named by ω and by $\omega + \omega$ have the same size. [2 points]

2. How does Brouwer think that his constructive conception of mathematics entails the rejection of the law of the excluded middle? [2 points]

3. Show how one can construct a counterexample to the law of the excluded middle by using Kripke models. Kripke models are not single models, but rather collections of 'nodes' (each a model), which are partially ordered by an 'accessibility' relation. In your argument, you can make use of the following three rules of Kripke models, for any proposition Φ:

   (I) Φ is true at node w only if Φ is true at any node w' accessible to w

   (II) $\neg\Phi$ is true at node w only if Φ is not true at any node w' accessible to w

   (III) $(\Phi \vee \Psi)$ is true at w only if either Φ is true at w or Ψ is

   [2 points]

4. We have now discussed all three major 'isms' in the history of philosophy of mathematics—logicism, formalism, and intuitionism. Which one do you find the most attractive, and why? What do you think is its major weakness? Give a sketch of how this weakness might be overcome. [4 points]